Iowa State University
Digital Repository

Retrospective Theses and Dissertations

Iowa State University Capstones, Theses and Dissertations

1955

Representation of power system loads for fault studies

G. Dale Sheckels
Iowa State College

Follow this and additional works at: https://lib.dr.iastate.edu/rtd

Part of the Electrical and Electronics Commons

Recommended Citation

Sheckels, G. Dale, "Representation of power system loads for fault studies " (1955). *Retrospective Theses and Dissertations*. 13291.
https://lib.dr.iastate.edu/rtd/13291

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

المنارة للاستشارات

www.manaraa.com

NOTE TO USERS

This reproduction is the best copy available.

UMI®

REPRESENTATION OF POWER SYSTEM LOADS
FOR FAULT STUDIES

by

G. Dale Sheckels

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

IOWA STATE COLLEGE LIBRARY AMES IA

Approved:

Signature was redacted for privacy.
In Charge of Major Work

Signature was redacted for privacy.
Head of Major Department

Signature was redacted for privacy.
Dean of Graduate College

Iowa State College

1955

UMI Number: DP12409

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI®

UMI Microform DP12409

Copyright 2005 by ProQuest Information and Learning Company.

All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

TK3226
Sh32r

1186-34

TABLE OF CONTENTS

T11725

INTRODUCTION

In stability studies of electric power systems, the system loads may be represented in several different ways with varying degrees of exactness and convenience.[1]

The simplest and crudest method is to neglect the loads completely. This method is restricted to fault studies where only the initial currents are desired for relay settings or momentary duty of circuit breakers. If induction motors or synchronous motors constitute a large portion of the load, then they cannot be neglected, but must be represented by their subtransient reactances in series with their pre-fault internal voltages.

Another method is to represent the loads by constant shunt impedances which are determined from the specified pre-fault active and reactive powers and terminal voltages. This method is commonly used for transient stability studies on a-c network analyzers. It does not take into account any possible changes in the equivalent impedance characteristics of the load resulting from voltage changes, nor does it include any of the inertia effects of the induction motor and synchronous motor portions of the load.

A third method is to represent the synchronous motor and induction motor portions of the load by their respective internal voltages in series with their transient reactances. If this method is used for a system set up on a network analyzer, then the loads must be simulated by generator units of the analyzer, and application of the method is limited by the number of generator units available.

This paper presents still another method of representing the loads, this time by adjusted constant impedances. This method retains the convenience of the constant impedance method mentioned above, but gives a more accurate representation during and shortly after the fault. The loads are divided into the three important types--induction motors, synchronous motors, and lighting and heating loads--and each type is discussed separately.

REVIEW OF LITERATURE

In 1929 Park and Bancker (10) presented a method of representing loads for a transient stability study. They divided the load into three equal parts--resistance, induction motor, and synchronous motor--and then combined the induction motor synchronous reactance, the synchronous motor transient reactance, and the adjacent generator transient reactance all in parallel. The combined reactance was then placed in series with the generator internal voltage.

O. G. C. Dahl (4) in 1938 presented curves showing the variation of active and reactive power with voltage for several different types of loads. Falls and Hobson (6) used this information in the choice of a constant impedance to represent loads in a transient stability study of a particular system. Here no attempt was made to include the inertia effects of the rotating machine portions of the loads.

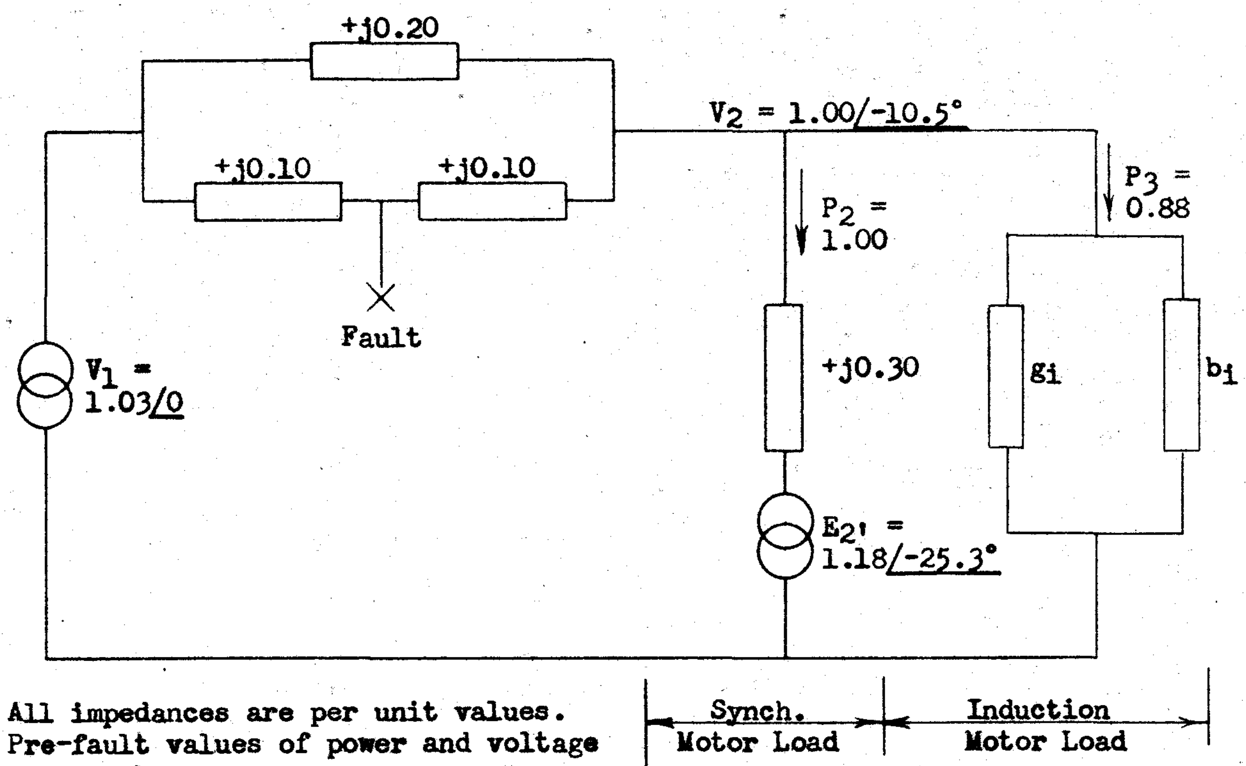
In a recent article by Shankle, Murphy, Long, and Harder (13), inertia effects were included in the representation of large induction motor loads. The induction motor was represented by its transient reactance in series with its pre-fault internal voltage. After the initial instant of the fault, this internal voltage was assumed to decay exponentially according to the short circuit time constant of the induction motor, and to be fixed spatially to the slipping rotor. In other respects the induction motor load was handled in a manner similar to that of a synchronous generator with negative power output.

INDUCTION MOTOR LOADS

Assumptions and Preliminary Derivations

In the analysis of the transient behavior of an induction motor and its effect on a power system, the first step was to assume an equivalent circuit of a simple hypothetical power system. This equivalent circuit is shown in figure 1 and is similar to the one used by Kimbark (7, p. 45). The following initial assumptions were made:

1. The sending end of the transmission line is connected to an infinite bus.
2. The receiving end load consists of a synchronous motor and an induction motor, initially of equal ratings and both operating at full load.
3. The synchronous motor voltage back of its transient reactance is a constant.
4. The synchronous motor damping is neglected.
5. The synchronous motor inertia constant H is 2.0 joules per volt-ampere.
6. Initially, the resistance was neglected in the transmission lines and in the synchronous motor. In the final circuit it is included.
7. A three-phase fault occurs at the middle of one of the transmission lines and is cleared 0.2 second later by the simultaneous opening of circuit breakers at both ends of the faulted line.
8. The decay of the magnetic field and the associated voltage in the



All impedances are per unit values.
Pre-fault values of power and voltage are shown.

Figure 1

Preliminary Power System Equivalent Circuit for Induction Motor Studies induction motor is so rapid that it has a negligible effect on the stability of the adjacent synchronous machine.

9. The load torque on the induction motor is a constant regardless of the slip.

10. Under-voltage protective devices have sufficient time lag to prevent operation during the low-voltage interval.

The induction motor load is the single motor equivalent of a large number of small motors ranging from 0.5 to 30 horse-power. The per unit parameters of these motors were calculated from data given by Puchstein, Lloyd and Conrad (11, p. 698) and are summarized in table 1. The averages of the per unit values were then used in a conventional exact equivalent circuit of the induction motor load and values were calculated for the equivalent admittance versus slip curves of figure 2 and for the synchronous torque versus slip curves of figure 3. In figure 2, admittance components were plotted rather than the more common impedance components as a convenience in later calculations. From table 1 the average full-load synchronous torque is 0.75 per unit and from figure 3 the corresponding slip at 1.00 per unit terminal voltage is 0.044. The equivalent conductance, g_i, of figure 2, corresponding to this slip, was then multiplied by the terminal voltage squared to give an induction motor power input of 0.88 per unit. These are the pre-fault operating conditions of the induction motor, and from these and the circuit constants of figure 1, the internal voltage of the synchronous motor was found. The magnitude of this voltage remains constant but its angle changes during the transient.

Table 1

Per Unit Parameters of
Small Induction Motors

HP	Synch. Speed	r_1	r_2	x_1 x_2	g_0	b_0	Full Load Torque
0.5	1800	0.0610	0.0470	0.0470	0.1016	0.763	0.547
1	1800	0.0691	0.0381	0.0535	0.0884	0.515	0.679
3	1800	0.0542	0.0375	0.0700	0.0638	0.404	0.752
5	1800	0.0472	0.0697	0.0820	0.0456	------	0.810
7.5	1800	0.0379	0.0335	0.0703	0.0405	0.314	0.838
10	1800	0.0500	0.0478	0.0795	0.0420	0.276	0.810
20	1800	0.0456	0.0394	0.0805	0.0432	0.238	0.842
30	1800	0.0354	0.0466	0.0831	0.0351	0.230	0.860
2	3600	0.0695	0.0419	0.0646	0.0654	0.345	0.742
2	1800	0.0532	0.0370	0.0617	0.0670	0.543	0.716
2	1200	0.0793	0.0570	0.0906	0.0614	0.636	0.633
2	900	0.0843	0.0558	0.1065	0.0599	0.718	0.575
2	720	0.0461	0.0828	0.1160	0.0710	0.710	0.599
Average		0.0564	0.0488	0.0774	0.0602	0.474	0.75

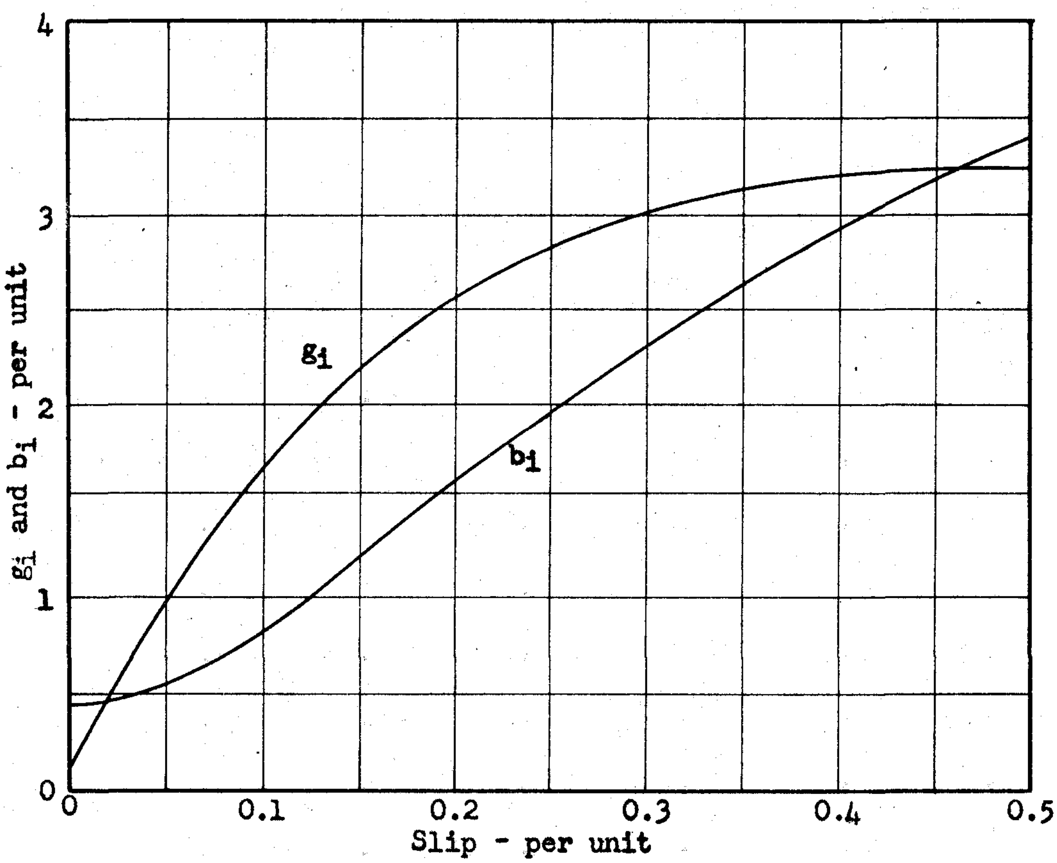


Figure 2

Equivalent Admittance
Components of Average
Induction Motor

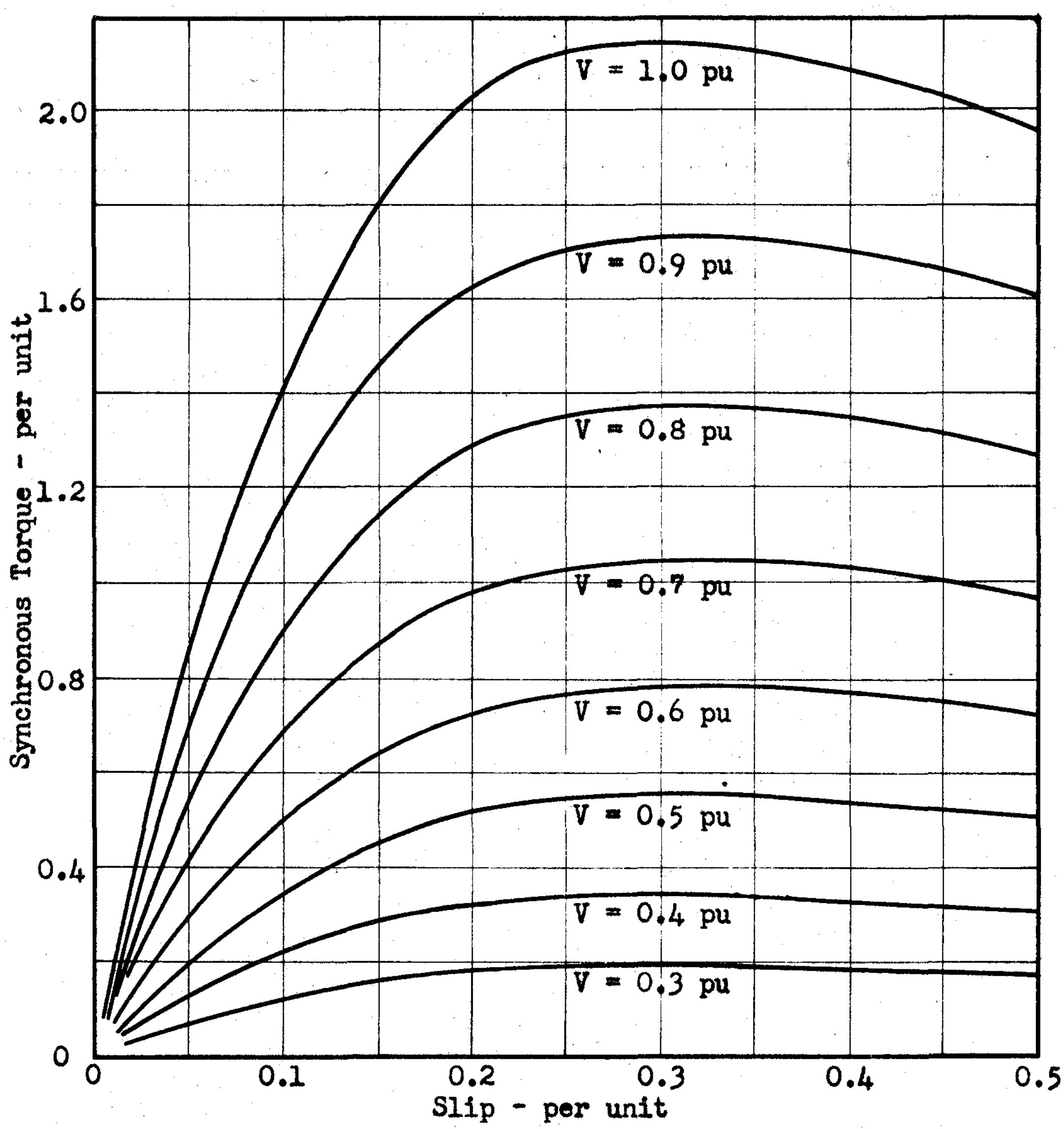


Figure 3

Synchronous Torque of
Average Induction Motor

After the fault occurs, the terminal voltage of the induction motor drops, its slip increases, and its equivalent admittance presented to the system increases according to the curves of figure 2. The acceleration is

$$\frac{d\omega}{dt} = \frac{T_M' - T_L'}{J}, \tag{1}$$

where ω = actual motor speed in radians per second,

T_M' = motor output torque in newton-meters,

T_L' = load torque in newton-meters,

J = motor and connected load inertia in newton-meter-seconds^2.

The actual speed is

$$\omega = \omega_s - s', \tag{2}$$

and

$$\frac{d\omega}{dt} = -\frac{ds'}{dt}, \tag{3}$$

where ω_s = synchronous speed in radians per second,

s' = slip in radians per second.

Substitution of equation (3) into equation (1) with a rearrangement gives

$$ds' = \frac{(T_L' - T_M')}{J}\,dt. \tag{4}$$

As a step-by-step process was to be used for the solution of this stability problem, equation (4) was written as

$$\Delta s' = \frac{(T_L' - T_M')}{J}(\Delta t), \tag{5}$$

where Δt = the time interval used in the step-by-step process,

$\Delta s'$ = change in slip during time interval Δt.

In order that the slip and torque might be used in per unit form, equation (5) was modified to give

$$\Delta\left(\frac{s'}{\omega s}\right) = \frac{\dfrac{(T_L' - T_M')(\omega_s)}{\text{Rating}}}{\left(\dfrac{\frac{1}{2} J \omega_s^2}{\text{Rating}}\right) 2} (\Delta t), \qquad (6)$$

or

$$\Delta s = \frac{(T_L - T_M)(\Delta t)}{2H}, \qquad (7)$$

where Δs = per unit change in slip

T_L = per unit load torque

T_M = per unit motor torque

H = inertia constant or ratio of the kinetic energy at rated speed to the machine volt-ampere rating.

Kimbark (7, p. 26) suggests an average value of H = 0.5 for an induction motor. This value was used for the initial studies but later it was varied to observe the effects of higher and lower inertia loads. The value of T_M was obtained from the curves of figure 3 for the corresponding slip and terminal voltage. As 0.05 second was to be used for Δt, the time interval, in the step-by-step process, equation (7) became

$$\Delta s = (T_L - T_M)(0.05). \qquad (8)$$

The slip at the end of the time interval is

$$s_n = s_{(n-1)} + \Delta s = s_{(n-1)} + 0.05(T_L - T_M)_{(n-1)} \qquad (9)$$

The equation for the swing angle δ of the synchronous motor as given by Kimbark is

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + \frac{180f}{GH} P_{a(n-1)}(\Delta t)^2 , \qquad (10)$$

where δ_n = swing angle at end of n'th time interval,

$\delta_{(n-1)}$ = swing angle at end of preceding interval,

$\Delta\delta_{(n-1)}$ = change in swing angle for preceding interval,

f = system frequency = 60 cps,

G = synchronous motor rating = 1 per unit,

$P_{a(n-1)}$ = accelerating power at end of preceding interval,

H = synchronous motor inertia constant = 2 joules/volt-ampere,

Δt = time interval = 0.05 second.

Inserting the above numerical values, equation (9) becomes

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 13.5 \ P_{a(n-1)}. \qquad (11)$$

Swing Curve Calculations

Equation (9) and (11) were used in the step-by-step calculations for the swing angle of the synchronous motor as shown in table 2. The resulting swing curve is shown in figure 4 and is labeled "exact curve". The entries in table 2 and the necessary calculations will now be explained:

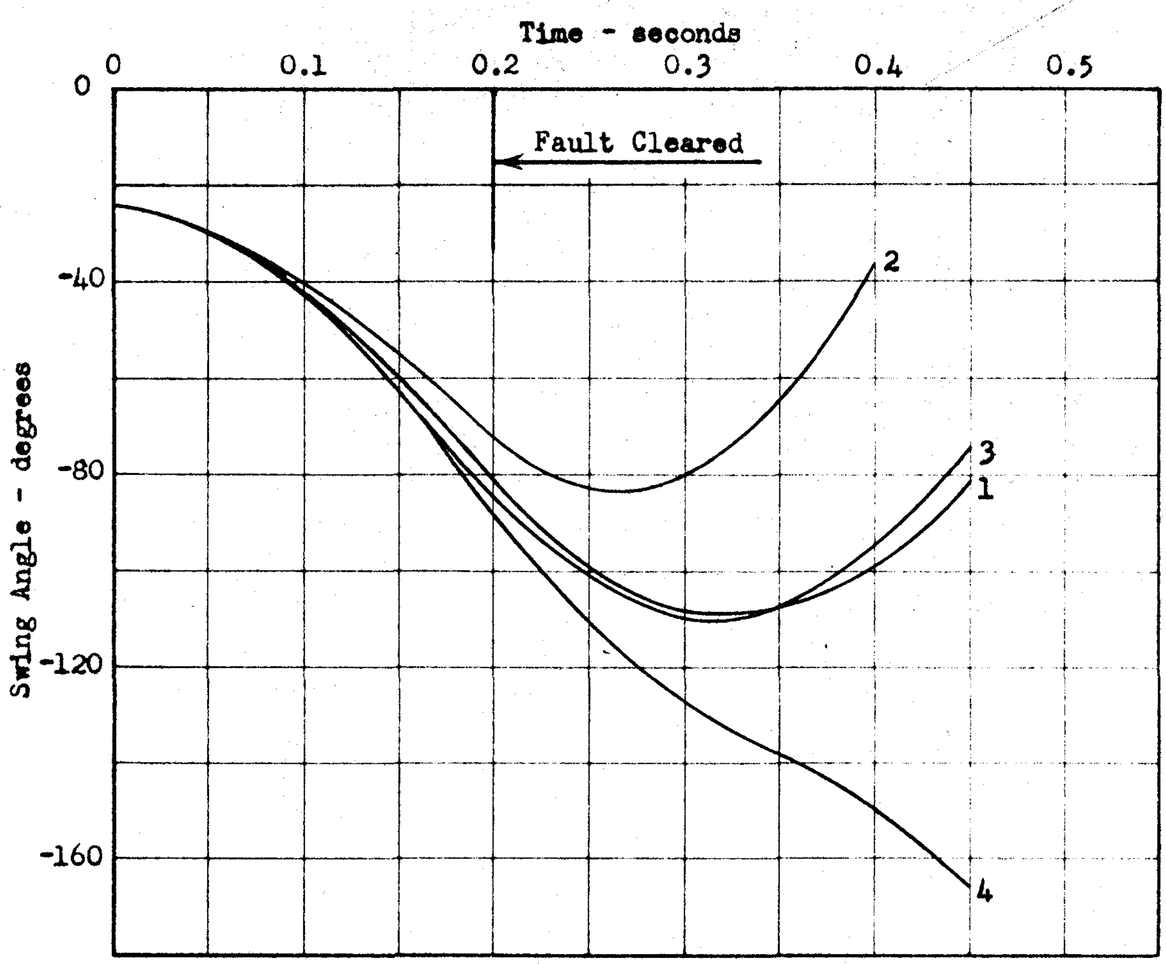
Column 1. The time interval is 0.05 second. The fault was applied at t = 0.00 and was removed at t = 0.20 by simultaneous opening of circuit breakers at both ends of the faulted line.

Column 2. The $g_i - jb_i$ is the induction motor equivalent admittance from

Table 2

Step-by-step Calculations for Synchronous Motor Swing Curve

1 Time	2 $g_i - jb_i$	3 $V_{2T}/\delta V_{2T}$	4 P_2	5 P_a	6 13.5 P_a	7 $\Delta\delta_2$	8 δ_2	9 T_M	10 $T_L - T_M$	11 Δs	12 s
0.00-	0.88-j0.51	1.00 /-10.5°	1.00	0.00			-25.3	0.75	0.00		0.044
0.00+	0.88-j0.51	0.467/-14.5	0.343	-0.657			-25.3	0.15	0.60		0.044
0.00A				-0.329	-4.4				0.30		
						- 4.4				0.015	
0.05	1.10-j0.58	0.463/-16.5	0.414	-0.586	-7.9		-29.7	0.19	0.56		0.059
						-12.3				0.028	
0.10	1.52-j0.72	0.442/-22.2	0.509	-0.491	-6.6		-41.0	0.26	0.49		0.087
						-18.9				0.025	
0.15	1.82-j0.92	0.402/ 31.0	0.762	-0.238	-3.2		-59.9	0.24	0.51		0.112
						-22.1				0.026	
0.20-	2.07-j1.12	0.348/-40.6	0.902	-0.098			-82.0	0.21	0.54		0.138
0.20+	2.07-j1.12	0.710/-46.6	1.612	+0.612			-82.0	0.87	-0.12		0.138
0.20A			1.257	+0.257	+3.5			0.54	0.21		
						-18.6				0.010	
0.25	2.17-j1.22	0.601/-53.9	1.718	+0.718	+9.5		-100.6	0.64	0.11		0.148
						- 9.1				0.006	
0.30	2.22-j1.26	0.543/-56.8	1.700	+0.700	+9.4		-109.7	0.55	0.20		0.154
						+ 0.3				0.010	
0.35	2.28-j1.34	0.538/-56.8	1.685	+0.685	+9.2		-109.4	0.55	0.20		0.164
						+ 9.5				0.010	
0.40	2.38-j1.42	0.585/-54.4	1.638	+0.638	+8.6		-99.9	0.66	0.09		0.174
						+18.1				0.005	
0.45	2.42-j1.45	0.683/-48.3	1.478	+0.478	+6.4		-81.8	0.92	-0.17		0.179
						+24.5				-0.009	
0.50							-57.3				0.170



1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent admittance equal to pre-fault value.
3 - Curve using constant induction motor equivalent admittance corresponding to a 116 per cent increase in pre-fault load torque.
4 - Exact curve using network analyzer.

Synchronous motor: G_s=1.00; H_s=2.0; load=100%.
Induction motor: G_i=1.00; H_i=0.5; load=100%.

Figure 4

Swing Curves for Synchronous Motor
with Parallel Induction Motor Load
Using Preliminary Equivalent Circuit

figure 3 corresponding to the slip at that particular instant. From previous calculations, the pre-fault slip and corresponding admittance were known. As the speed or slip of the induction motor cannot change instantly, the values of slip and admittance just after the fault remained the same as those just before the fault.

Column 3. The receiving end terminal voltage V_2 was found by an application of the superposition theorem. First, the synchronous motor internal voltage E_2 was removed and the component of V_2 caused by V_1 was calculated. Then E_2 was restored, V_1 was removed, and the second component of V_2 calculated. The actual V_2 was the complex sum of the two components.

Column 4. P_2 is the power input to the synchronous motor and was found from the following equation:

$$P_2 = \frac{V_2 E_2}{X_d'} \sin(\delta_{V_2} - \delta_2), \qquad (12)$$

where X_d' = synchronous motor transient reactance = 0.30

E_2 = synchronous motor internal voltage = 1.178

δ_{V_2} = angle of V_2 with respect to V_1 as a reference

δ_2 = angle of E_2 with respect to V_1 as a reference.

Since it was assumed that the synchronous motor resistance was zero, its mechanical power output will also be equal to P_2.

Column 5. The accelerating power P_a is the excess of the mechanical power output P_2 over the load requirement which is 1.00 per unit.

Columns 6, 7, and 8. The swing angle δ was found by application of equation (11).

Column 9. Figure 3 was used to find the induction motor output torque T_M corresponding to the terminal voltage V_2 and the slip s at the particular instant.

Column 10. The decelerating torque is the difference between the torque requirement of the load and the output torque of the motor. At the switching instants, t = 0.00 and t = 0.20, the decelerating torque was taken as the average of values existing just before and just after the switching instant.

Columns 11 and 12. The slip at the end of the interval or at the beginning of the next interval was found from equation (9). The new value of slip was then used to find the induction motor equivalent admittance and torque for the beginning of the next interval.

The exact swing curve for the synchronous motor resulting from the calculations is shown in figure 4 and is labeled no. 1. The curve is not truly exact in that the method contains approximations and its accuracy is dependent on the length of the time intervals used. However, this exact curve is produced by what is believed to be the most accurate method available and it is used as a base for comparison of the more approximate methods.

If a constant impedance or admittance is used to represent the induction motor, the problem is simplified considerably. It is no longer necessary to find the value of V_2 for each step nor to even consider the inertia effects of the induction motor, and the problem reduces to the simple two-machine case with a shunt load. The method of handling this simpler problem is given in any transient stability text, such as Kimbark, but for completeness, it is again developed using the nomenclature of this particular problem.

The circuit of figure 1, with the constant admittance replacing the induction motor, is reduced by parallel combinations and a wye-delta transformation to that of figure 5. Only the identities of the original V_1 and E_2 are retained. From the circuit of figure 5, the current flowing into the synchronous motor is

$$I_2 = (V_1 - E_2)Y_{12} - E_2 Y_{20}. \tag{13}$$

The power input to the synchronous motor is

$$P_2 = \text{Real} \left[\bar{E}_2 I_2 \right], \tag{14}$$

where $\bar{E}_2$ = conjugate of E_2, the synchronous motor internal voltage. After substitution of equation (13) into equation (14), the real part is taken and P_2 becomes

$$P_2 = V_1 E_2 Y_{12} \cos(\theta_{12} - \delta_2) - E_2^2 Y_{22} \cos \theta_{22}, \tag{15}$$

where θ_{12} = angle of Y_{12},

δ_2 = angle of E_2, the swing angle,

Y_{22} = magnitude of the complex sum of Y_{12} and Y_{20},

θ_{22} = angle of Y_{22}.

For the case in which the pre-fault admittance of the induction motor was used in the equivalent circuit, the numerical step-by-step calculations are shown in table 3 and the resulting swing curve in figure 4. An explanation of table 3 follows:

Column 2. The angle θ_{12} is associated with the admittance Y_{12} of the

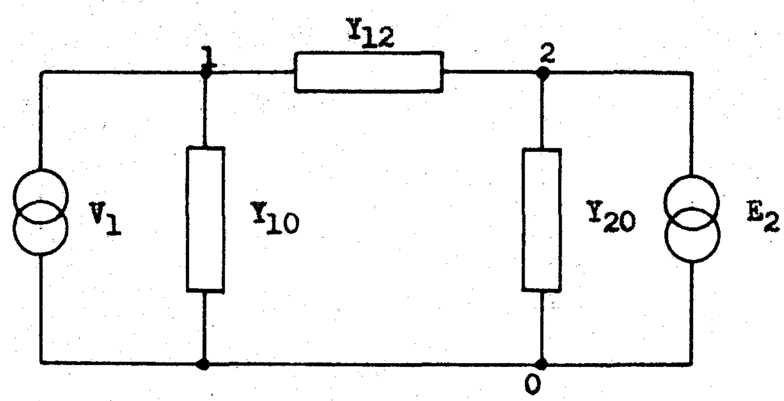


Figure 5

Power System Equivalent Circuit
for Constant Impedance Loads

Table 3

Step-by-step Calculations for
Synchronous Motor Swing Curve

Induction Motor Load Represented
by a Constant Impedance

1	2	3	4	5	6	7	8	9	10	11	12
Time	θ_{12}	$\theta_{12}-\delta_2$	cos $\theta_{12}-\delta_2$	P_{12m}	P_{12}	P_{22}	P_2	$+P_2-1 = P_a$	13.5 P_a	$\Delta\delta_2$	δ_2
0.00-							+1.000	0			-25.26
0.00+	-92.68	-67.42	+0.383	+1.071	+0.414	0	+0.414				-25.26
0.00A							+0.707	-0.293	- 3.94		-25.26
										- 3.94	
0.05		-63.48	+0.446		+0.478		+0.478	-0.522	- 7.04		-29.20
										-10.98	
0.10		-52.50	+0.609		+0.652		+0.652	-0.348	- 4.69		-40.18
										-15.67	
0.15		-36.83	+0.800		+0.857		+0.857	-0.143	- 1.92		-55.58
										-17.59	
0.20-		-19.24	+0.944		+1.01		+1.01				-73.44
0.20+	-95.72	-22.28	+0.925	+2.28	+2.11	+0.17	+1.94				-73.44
0.20A							+1.47	+0.47	+ 6.33		-73.44
										-11.23	
0.25		-11.05	+0.981		+2.24		+2.07	+1.07	+14.40		-84.67
										+ 3.17	
0.30		-14.22	+0.969		+2.21		+2.04	+1.04	+14.00		-81.50
										+17.17	
0.35		-31.39	+0.854		+1.95		+1.78	+0.78	+10.50		-64.33
										+27.67	
0.40											-36.66

equivalent circuit of figure 5. It is constant for the duration of the fault, and then changes to a new constant value after the fault is cleared.

Column 5. P_{12m} is the constant coefficient of the first cosine term of equation (15).

Column 6. P_{12} is the product of P_{12m} from column 5 and the cosine factor from column 4. It is the complete first term of equation (15).

Column 7. P_{22} is the complete second term of equation (15). It has two constant values, one during the fault and a second after the fault is cleared.

Column 8. P_2 is the input or output power of the synchronous motor and is equal to column 6 minus column 7 according to equation (15).

Column 9. P_a is the accelerating power or the power output minus load requirement. The load requirement is constant at 1.00 per unit.

Columns 10, 11, and 12. The swing angle δ was found by an application of equation (11).

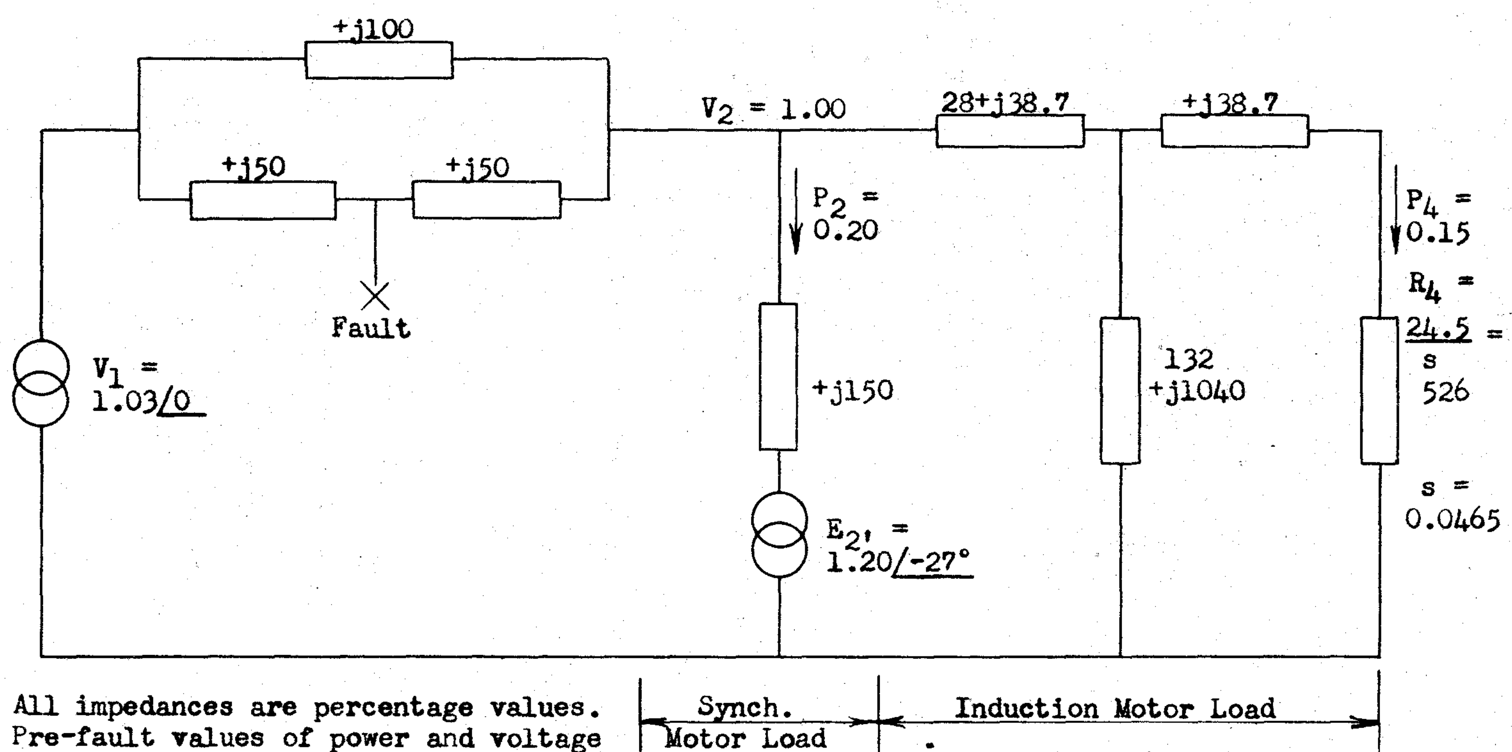
A comparison of the exact curve and the pre-fault admittance curve of figure 4 shows that the commonly used pre-fault equivalent of the induction motor gives a swing curve with too low an amplitude. Several larger values of induction motor admittance were tried and a value of $g_1 - jb_1 = 2.00 - j1.00$ appeared to give the closest match to the exact curve. This curve is also shown in figure 4. From figures 2 and 3 the values of slip and torque corresponding to this admittance are about 0.125 and 1.62 respectively. In other words, for this particular case an increase in induction motor loading of 116 per cent immediately after the fault would give a synchronous motor swing curve approximating the exact curve.

Swing Curves from Network Analyzer Measurements

The actual long-hand calculations for these curves proved to be quite laborious and time consuming. For these reasons, most of the succeeding studies were made on the network analyzer. The equivalent circuit used on the network analyzer is shown in figure 6 and differs from the original equivalent circuit of figure 1 in several respects.

First, the current level was reduced by a factor of five in order to bring the currents of this particular problem within the ratings of the network analyzer elements. This necessitated a corresponding increase in the impedance level and reduction in power and torque levels by the same factor of five. The numerical coefficients of the torque term of equation (9) and the power term of equation (11) were increased by the same factor to give the original values of slip and delta angle. The voltage level remained the same.

The second difference was the substitution of the exact equivalent circuit of the induction motor for the parallel g_i and b_i of figure 1. This substituted circuit permitted a direct measurement of the induction motor air-gap power P_4, which is numerically equal to the per unit developed torque T_M.



All impedances are percentage values.
Pre-fault values of power and voltage are shown.

Figure 6

Preliminary Network Analyzer
Power System Equivalent Circuit
for Induction Motor Studies

The step-by-step calculations used with the network analyzer to obtain a solution to this problem were quite similar to those explained previously. Therefore, only the differences will be explained here. The induction motor slip was calculated from the measured motor torque and load torque in a manner similar to that previously explained, except that equation (9) was modified slightly by substitution of the new numerical coefficient, as

$$s_n = s_{(n-1)} + 0.25(T_L - T_M)_{(n-1)}. \qquad (9')$$

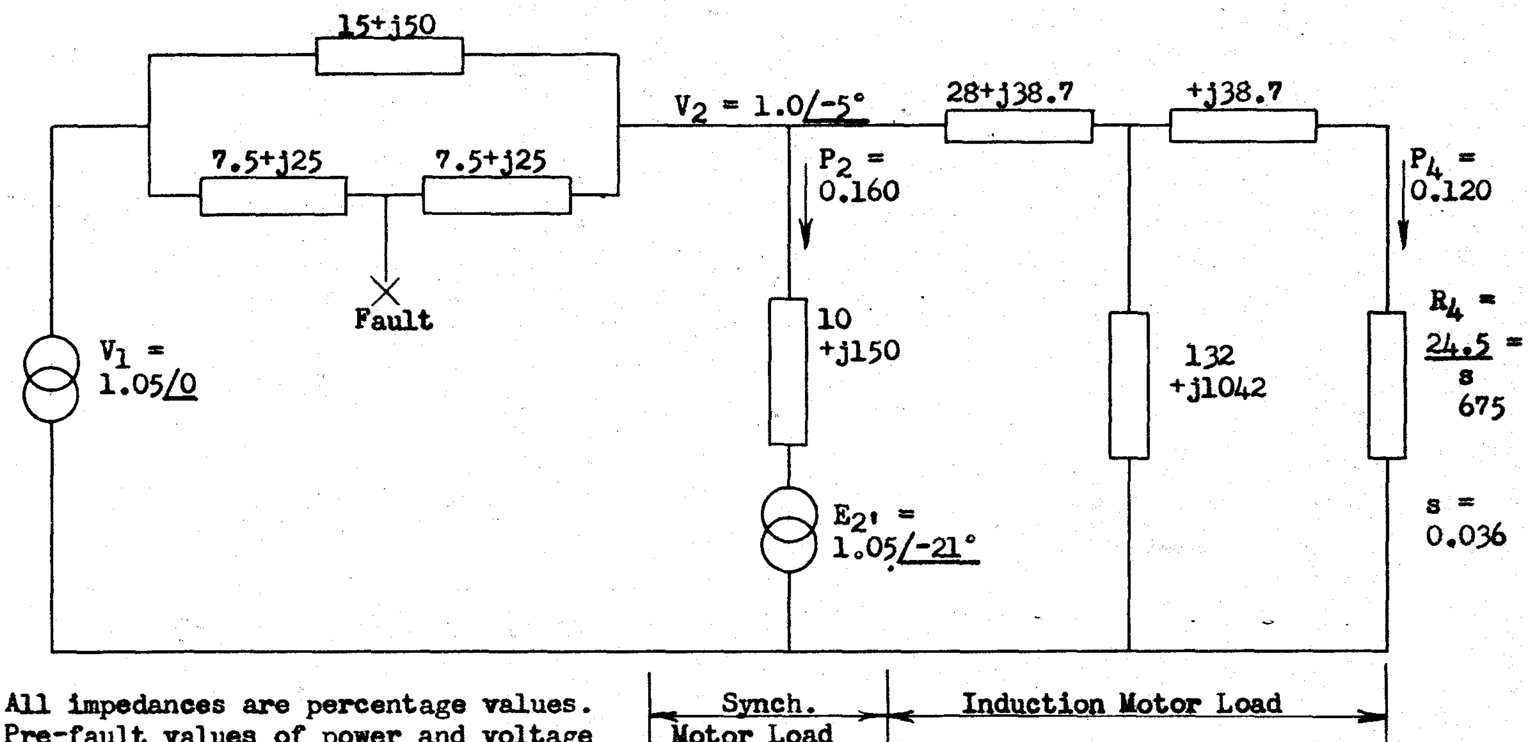
This value of slip was then used to find the new value of R_4 in figure 6 which was to exist at the beginning of the next interval. The synchronous motor power was measured directly as P_2, and, as before, it was used to find the delta angle by application of the modified form of equation (11), as

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 67.5\, P_{a(n-1)}. \qquad (11')$$

The angle of E_2 was adjusted to this new value of δ to represent the synchronous motor conditions existing at the beginning of the next interval. The powers P_4 and P_2 were again measured and the calculations were repeated for each succeeding time interval.

The curve for the above calculations is no. 4 in figure 4, and from its shape it indicates that the synchronous motor is unstable and is pulling out of step. Since this curve is not at all like the exact curve, some discrepancy must exist between the network analyzer method and the analytical or long-hand method. In the analytical method, resistances in the lines and in the synchronous motor were assumed to be zero, but in the network analyzer method, the small inherent resistances of the reactance units and connections were included in the equivalent circuit. This difference in the equivalent circuits explains the non-similarity of the two curves. Since it was impossible to eliminate the inherent resistances from the network analyzer, the only other alternative was to change the specifications of the original equivalent circuit to include some resistance.

Resistance values of 30 per cent and 10 per cent were initially assumed for each of the two transmission lines and for the synchronous motor, respectively, but the settings actually used on the network analyzer were slightly smaller than these by the values of the inherent resistances. A network analyser solution of this new problem showed that the system was still unstable. Since it was desired to have the system just on the point of instability, the problem specifications were again modified. First, the voltage V_1 was increased to 1.10 per unit, but the system was still unstable. Further changes were then made, each followed by a solution of the problem, until finally the circuit of figure 7 was evolved, which was just slightly on the unstable side of the critical point. In this equivalent circuit, the V_1 has been changed back to 1.05 per unit; the transmission line impedances have been halved, and the initial loads on both the induction motor and synchronous motor have been reduced to 80 per cent of their original full load values.



All impedances are percentage values.
Pre-fault values of power and voltage
are shown.

Figure 7

Network Analyzer Power System Equivalent
Circuit for Induction Motor Studies

$$\frac{\text{Induction Motor Rating}}{\text{Synchronous Motor Rating}} = 1$$

Transient Studies with Induction Motor Rating Equal to Synchronous Motor Rating

The calculations for the exact curve for this circuit are summarized in table 4 and the exact curve itself is shown in figure 8. The calculations using the constant pre-fault induction motor impedance are summarized in table 5 and the resulting approximate curve is also shown in figure 8. The remaining two curves of figure 8 were obtained in a similar manner. It will be noticed that the curve for the pre-fault induction motor impedance indicates a stable rather than unstable condition which is a considerable deviation from the exact curve. The other two approximate curves use induction motor impedances corresponding to increases of load torque of 85 per cent and 115 per cent. They bracket the exact curve and from their position it may be estimated that an increase in load torque of 100 per cent would give a curve approximating the exact curve.

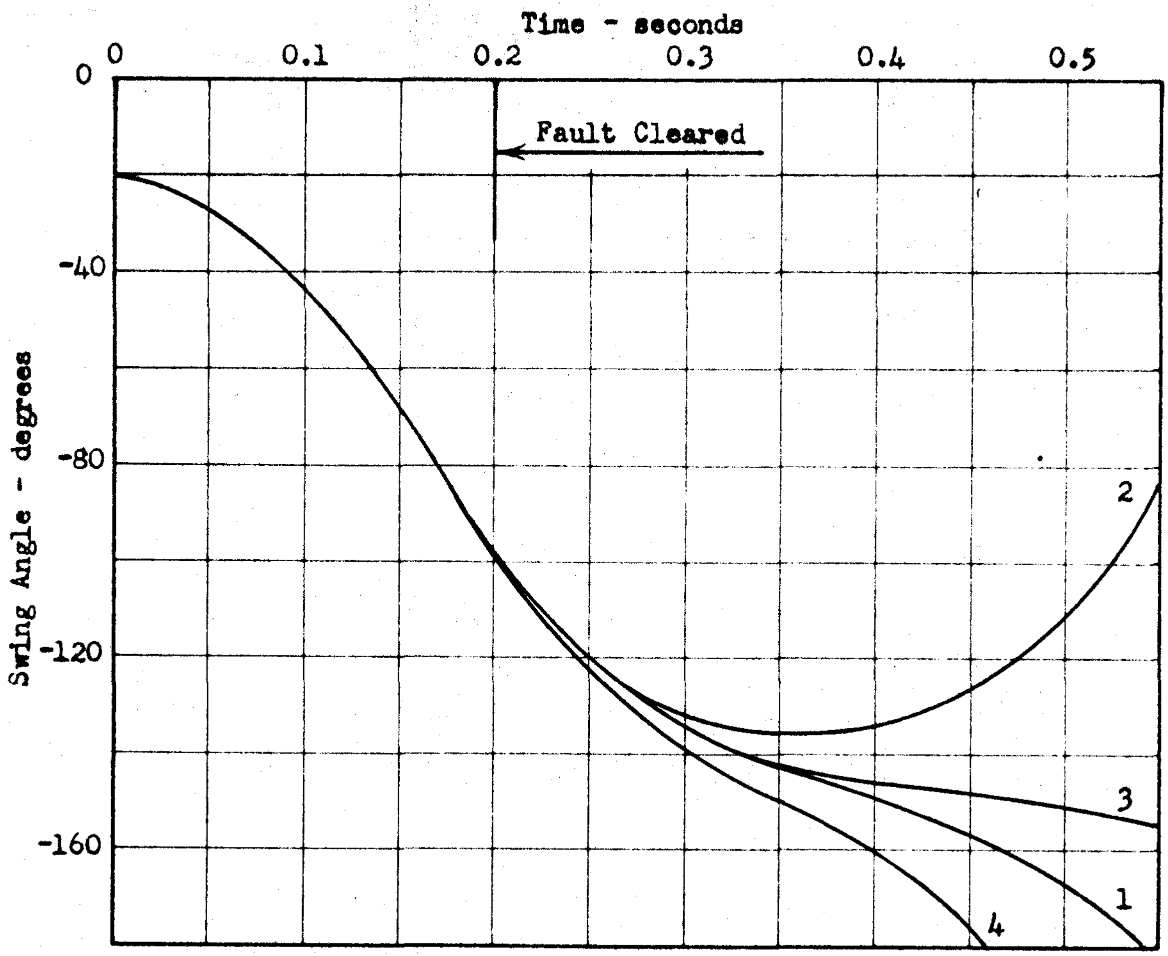
A similar set of calculations were made for an initial loading of 75 per cent on both induction motor and synchronous motor, and the resulting curves are shown in figure 9. Again, the curve corresponding to a 100 per cent increase in load torque of the induction motor seems to approximate the exact curve, although here the shape of the curve is less sensitive to changes of torque.

With the initial loading returned to 80 per cent of full load, the induction motor inertia was increased by a factor of four to give an H of 2.0, and the curves of figure 10 were obtained. The pre-fault impedance curve was transferred from figure 8 since the pre-fault conditions were

Table 4

Step-by-step Calculations for Synchronous Motor Swing Curve Utilizing Network Analyzer

Time	P_2	$P_2-0.155 = P_a$	$67.5P_a$	$\Delta\delta_2$	δ_2	T_M	$0.12-T_M$	$\frac{0.12-T_M}{4} = \Delta s$	s	$\frac{24.5}{s}$
0.00-	+0.155	0			- 21.0	0.120	0		0.036	675
0.00+	-0.020				- 21.0	0.020			0.036	675
0.00A	+0.067	-0.088	- 5.9		- 21.0	0.070	0.050		0.036	675
				- 5.9				0.013		
0.05	-0.005	-0.166	-11.2		- 26.9	0.026	0.094		0.049	500
				-17.1				0.024		
0.10	+0.040	-0.115	- 7.7		- 44.0	0.033	0.087		0.073	336
				-24.8				0.022		
0.15	+0.090	-0.065	- 4.4		- 68.8	0.033	0.087		0.095	258
				-29.2				0.022		
0.20-	+0.110				- 98.0	0.026			0.117	210
0.20+	+0.360				- 98.0	0.135			0.117	210
0.20A	+0.235	+0.080	+ 5.4		- 98.0	0.081	0.039		0.117	210
				-23.8				0.010		
0.25	+0.300	+0.145	+ 9.8		-121.8	0.102	0.018		0.127	193
				-14.0				0.004		
0.30	+0.240	+0.085	+ 5.7		-135.8	0.085	0.035		0.131	187
				- 8.3				0.009		
0.35	+0.180	+0.025	+ 1.7		-144.1	0.077	0.043		0.140	175
				- 6.6				0.011		
0.40	+0.145	-0.010	- 0.7		-150.7	0.073	0.047		0.151	162
				- 7.3				0.012		
0.45	+0.100	-0.055	- 3.7		-158.0	0.070	0.050		0.163	150
				-11.0				0.013		
0.50					-169.0				0.176	



1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance corresponding to the pre-fault load torque.
3 - Curve using constant induction motor equivalent impedance corresponding to an 85 per cent increase in pre-fault load torque.
4 - Curve using constant induction motor equivalent impedance corresponding to a 115 per cent increase in pre-fault load torque.

Synchronous motor: G_s=1.00; H_s=2.0; load=80%.
Induction motor: G_i=1.00; H_i=0.5; load=80%.

Figure 8

Swing Curves for Synchronous Motor with Parallel Induction Motor Load

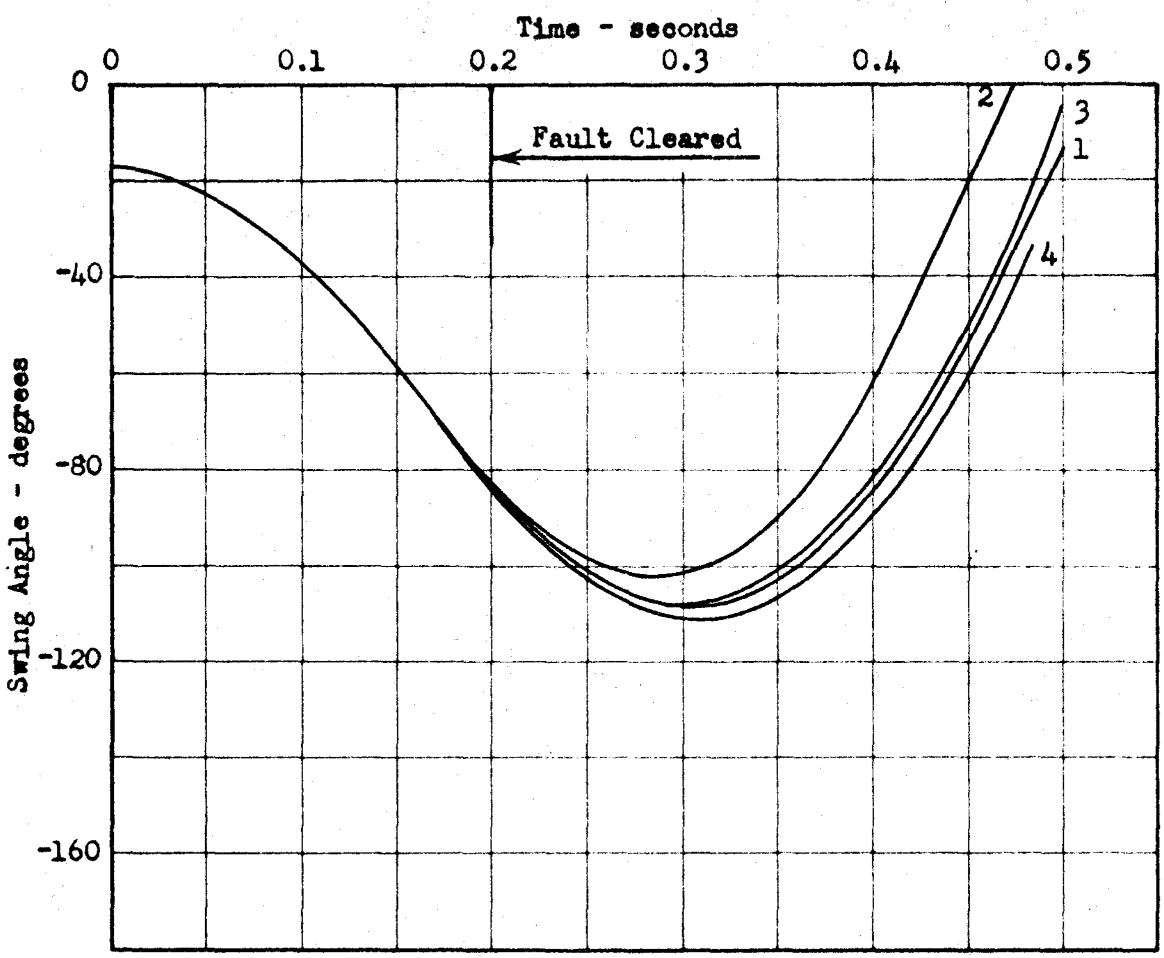
Table 5

Step-by-step Calculations for Synchronous Motor Swing Curve Utilizing Network Analyzer

Induction Motor Load Represented by its Pre-fault Impedance

Pre-fault slip = 0.036; $R_4 = \frac{24.5}{s} = 675\%$

Time	P_2	P_a	$67.5P_a$	$\Delta\delta_2$	δ_2
0.00-	+0.155	0.00			- 21.0
0.00+	-0.025				- 21.0
0.00A	+0.065	-0.090	- 6.1		- 21.0
				- 6.1	
0.05	-0.010	-0.165	-11.1		- 27.1
				-17.2	
0.10	+0.040	-0.115	- 7.7		- 44.3
				-24.9	
0.15	+0.095	-0.060	- 4.0		- 69.2
				-28.9	
0.20-	+0.110				- 98.1
0.20+	+0.390				- 98.1
0.20A	+0.250	+0.095	+ 6.4		- 98.1
				-22.5	
0.25	+0.320	+0.165	+11.1		-120.6
				-11.4	
0.30	+0.270	+0.105	+ 7.1		-132.0
				- 4.3	
0.35	+0.235	+0.080	+ 5.4		-136.3
				+ 1.1	
0.40	+0.245	+0.090	+ 6.1		-135.2
				+ 7.2	
0.45	+0.290	+0.135	+ 9.1		-128.0
				+16.3	
0.50	+0.360	+0.205	+13.8		-111.7
				+30.1	
0.55	+0.390	+0.235	+15.9		- 81.6
				+56.0	
0.60					- 25.6

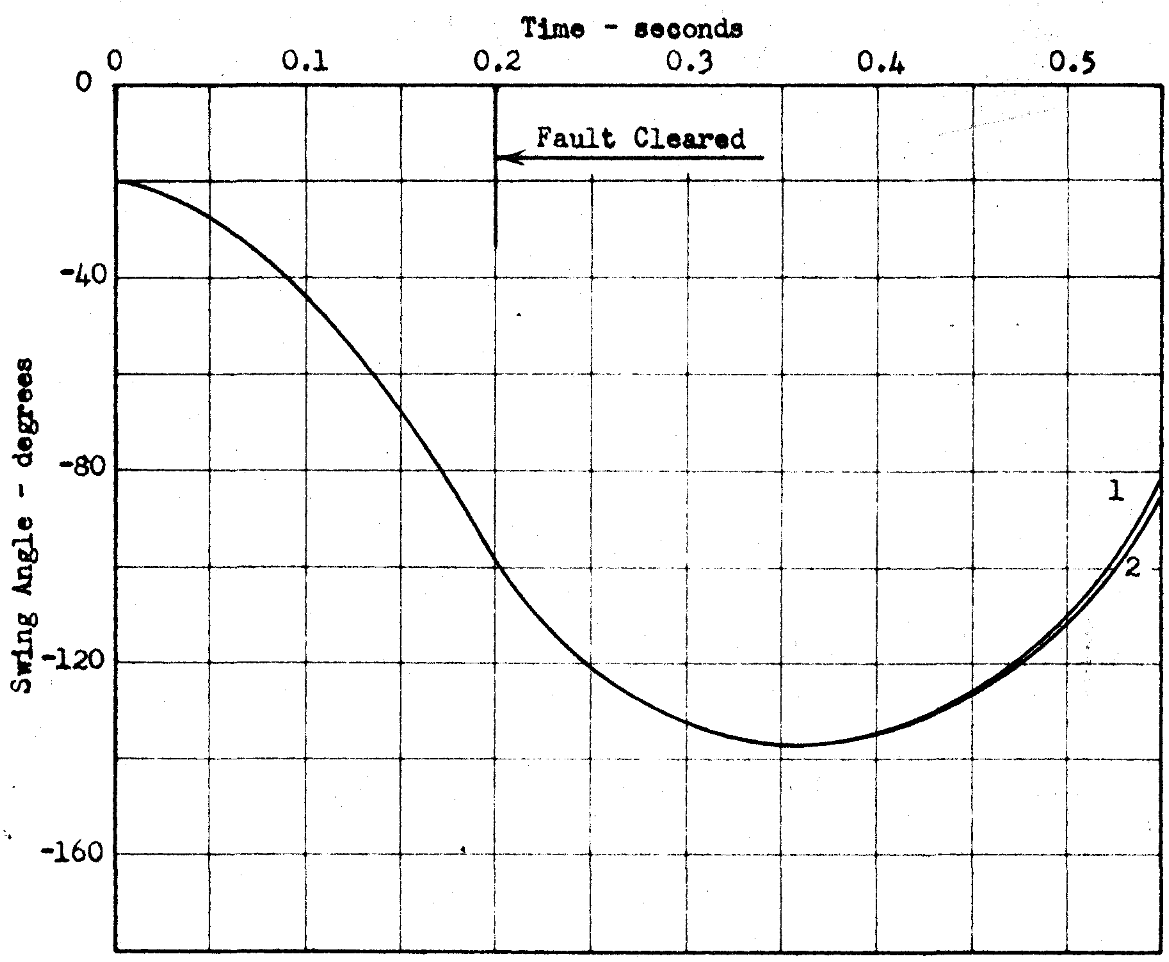


1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance equal to pre-fault value.
3 - Curve using constant induction motor equivalent impedance corresponding to an 85 per cent increase in pre-fault load torque.
4 - Curve using constant induction motor equivalent impedance corresponding to a 115 per cent increase in pre-fault load torque.

Synchronous motor: G_s=1.00; H_s=2.0; load=75%.
Induction motor: G_i=1.00; H_i= 0.5;load=75%.

Figure 9

Swing Curves for Synchronous Motor with Parallel Induction Motor Load



1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance equal to pre-fault value.

Synchronous motor: G_s=1.00; H_s=2.0; load=80%.
Induction motor: G_i=1.00; H_i=2.0: load=80%.

Figure 10

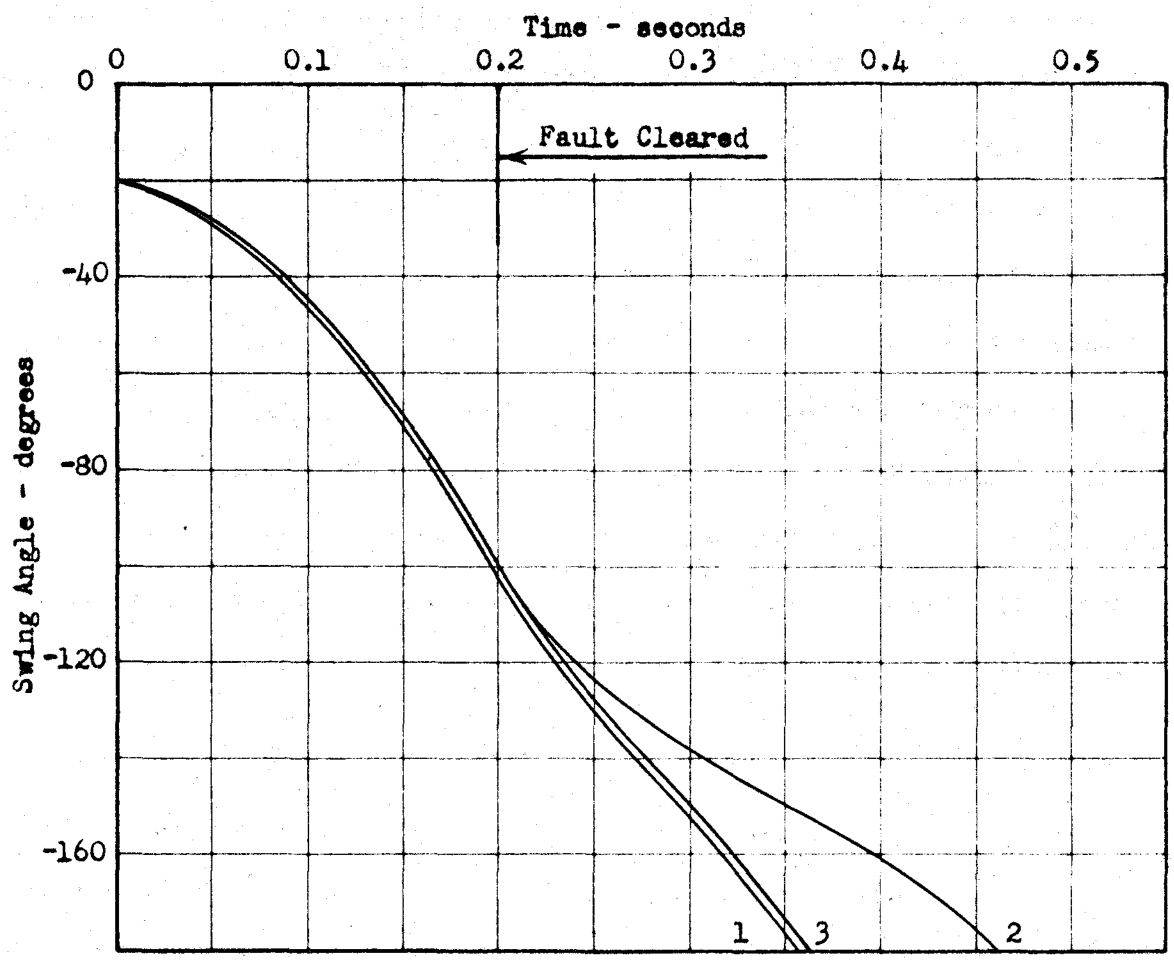
Swing Curves for Synchronous Motor with Parallel Induction Motor Load

identical in the two cases. The calculations for the exact curve are similar to those of table 4 but differ in the coefficient used in finding the Δs term. Here it is one-fourth its original value or 0.0625. It will be noticed from figure 10 that the exact and the pre-fault impedance curves are practically identical. This indicates that the common method of representing the induction motor load by its pre-fault impedance is accurate when the induction motor inertia is large. This seems logical for the impedance of the induction motor is a function of the slip only, and as the slip of the high inertia induction motor changed only slightly, from a pre-fault value of 0.036 to a maximum of 0.079, the actual impedance of the motor also changed only slightly from its pre-fault value.

Three more sets of calculations for an induction motor inertia constant of 0.125 result in the curves of figure 11. Here the necessary increase in load torque appears to be about 230 per cent. For this value of inertia constant, the slip of the induction motor increased from its pre-fault value of 0.036 to a value of 0.450 at 0.35 second and was still increasing at that time.

Transient Studies with Induction Motor Rating Equal to One-fourth of Synchronous Motor Rating

In order to study the effects of a smaller total installed capacity of induction motors, the rating of the equivalent induction motor was reduced to one-fourth of its original value, or to 0.05 per unit on the network analyzer base. This required an increase in the induction motor per

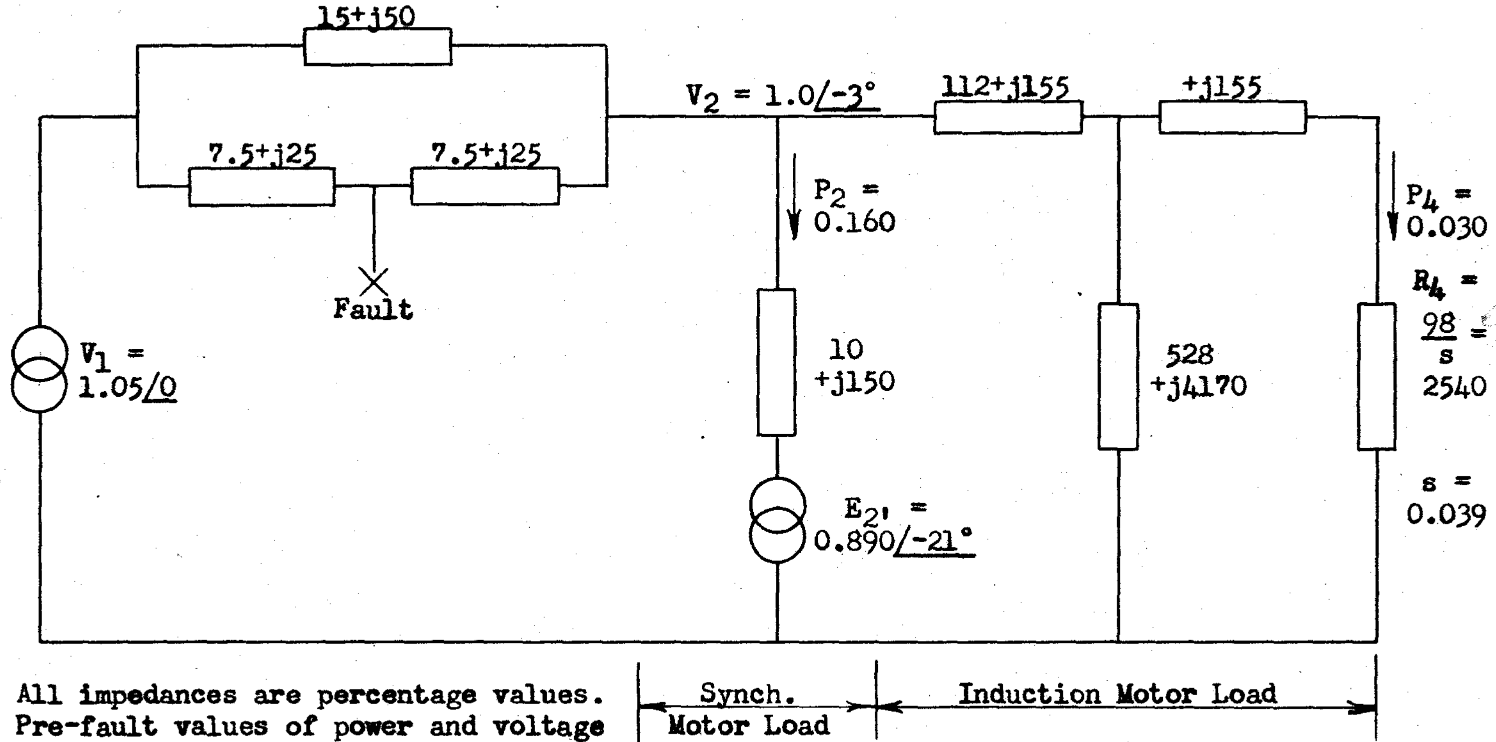


1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance corresponding to a 115 per cent increase in pre-fault load torque.
3 - Curve using constant induction motor equivalent impedance corresponding to a 230 per cent increase in pre-fault load torque.

Synchronous motor: G_s=1.00; H_s= 2.0; load=80%.
Induction motor: G_i=1.00; H_i=0.125; load=80%.

Figure 11

Swing Curves for Synchronous Motor with Parallel Induction Motor Load

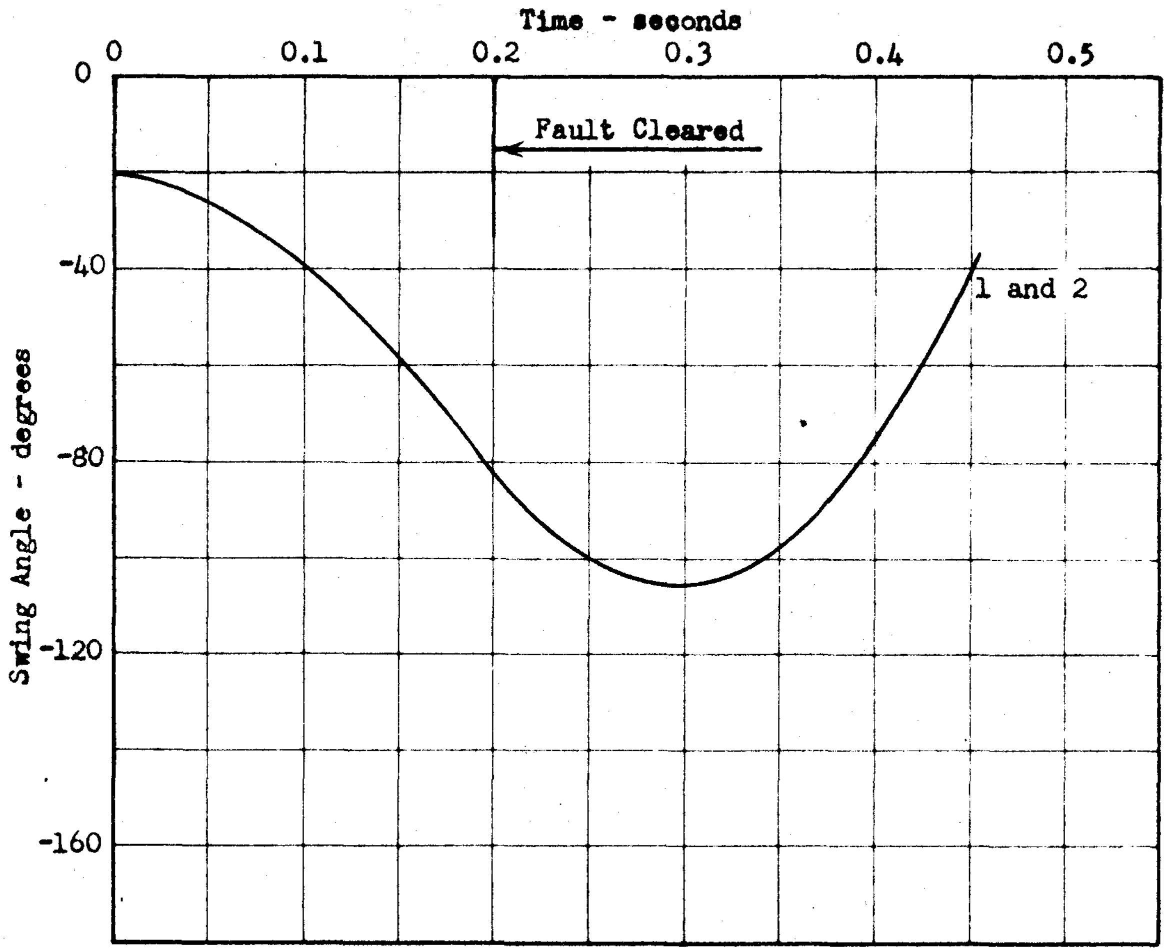


All impedances are percentage values.
Pre-fault values of power and voltage
are shown.

Figure 12

**Network Analyzer Power System Equivalent
Circuit for Induction Motor Studies**

$$\frac{\text{Induction Motor Rating}}{\text{Synchronous Motor Rating}} = \frac{1}{4}$$

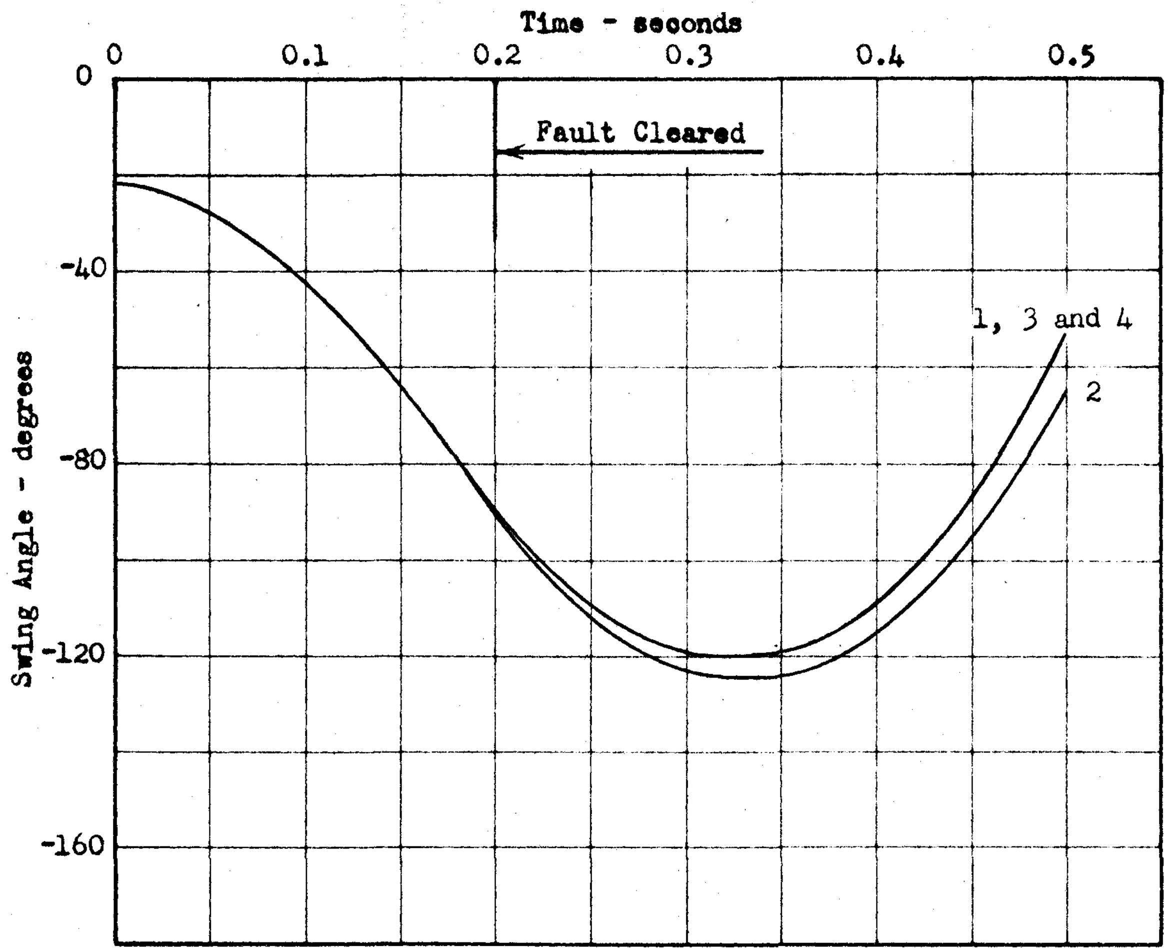


1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance equal to pre-fault value.

Synchronous motor: $G_s=1.00$; $H_s=2.0$; load=80%.
Induction motor; $G_i=0.25$; $H_i=2.0$; load=80%.

Figure 13

Swing Curves for Synchronous Motor with Parallel Induction Motor Load

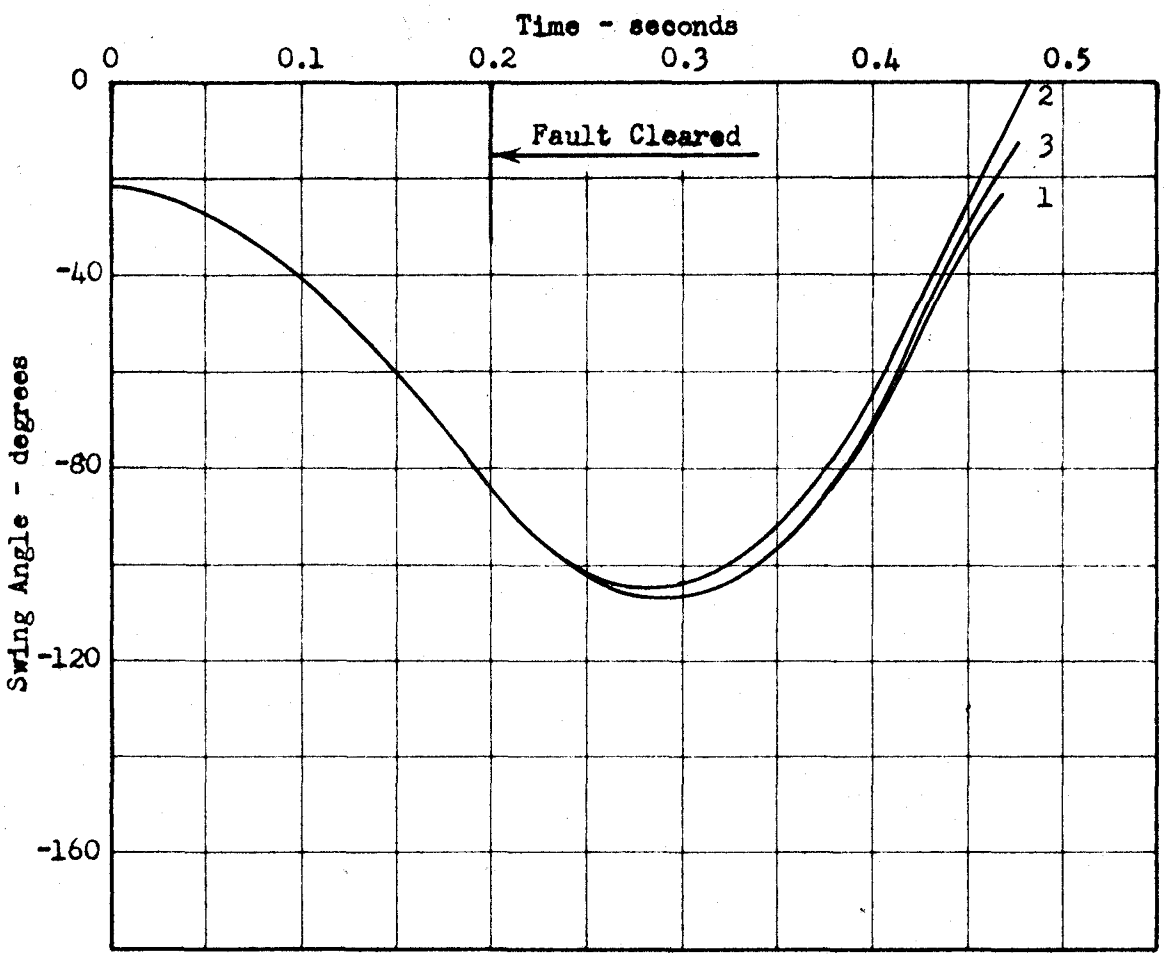


1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance equal to pre-fault value.
3 - Curve using constant induction motor equivalent impedance corresponding to a 97 per cent increase in pre-fault load torque.
4 - Curve using constant induction motor equivalent impedance corresponding to a 126 per cent increase in pre-fault load torque.

Synchronous motor: G_s=1.00; H_s=2.0; load=80%.
Induction motor: G_i=0.25; H_i=0.5; load=80%.

Figure 14

Swing Curves for Synchronous Motor with Parallel Induction Motor Load



1 - Exact swing curve for synchronous motor.
2 - Curve using constant induction motor equivalent impedance corresponding to a 238 per cent increase in pre-fault load torque.
3 - Curve using constant induction motor equivalent impedance corresponding to a 250 per cent increase in pre-fault load torque.

Synchronous motor: G_s=1.00; H_s= 2.0; load=80%.
Induction motor: G_i=0.25; H_i= 0.125; load=80%.

Figure 15

Swing Curves for Synchronous Motor with Parallel Induction Motor Load

unit impedances by a factor of four and a corresponding reduction of the initial power and torque by the same factor of four. The new equivalent circuit is shown in figure 12 along with the new pre-fault or initial conditions. The calculations were then repeated for induction motor inertia constants of 2.0, 0.50, and 0.125 with the resulting curves being shown in figures 13, 14, and 15. The induction motor pre-fault slip was 0.039 for all three inertias and the maximum slips were 0.066, 0.134, and 0.307, respectively, for the three studies. From these curves it is apparent that the pre-fault induction motor impedance will again give accurate results for the high inertia induction motors. For an induction motor inertia constant of 0.50, the necessary increase in induction motor load torque is between 97 per cent and 124 per cent, but is not at all critical. For the low inertia induction motors, the necessary load torque increase is about 250 per cent but again it is not critical.

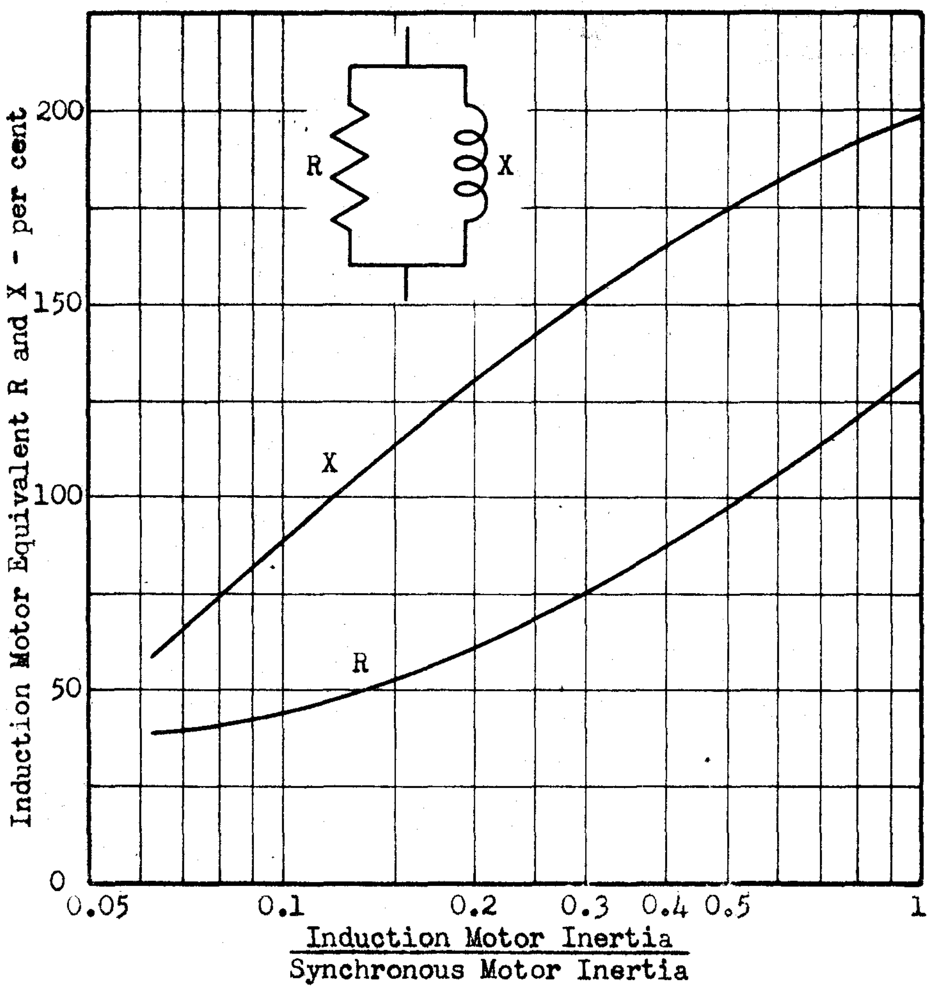
Results

The results of the last six cases are summarized in table 6. It should be noted from this table that the results of the $G_1 = 1.0$ cases can be used in the $G_1 = 0.25$ cases with only a slight loss of accuracy in the last case. If this is done, the combined results can be presented in the form of curves as in figure 16. For convenience in network analyzer setups, the ordinates of these curves are plotted in terms of the parallel equivalent resistances and reactances corresponding to the loading conditions of table 6. These resistances and reactances are percentage values referred to the induction motor's own base impedance of 100 per cent or

Table 6

Necessary Increases in Pre-fault Load Torque for a Constant Impedance Representation of Induction Motor Load

G_1	$H_1 = 2.0$	$H_1 = 0.5$	$H_1 = 0.125$
1.00	0	100%	230%
0.25	0	97 - 124	250



Range of $\frac{\text{Induction Motor Rating}}{\text{Synchronous Motor Rating}}$ = 0.25 to 1.00

Figure 16

Equivalent Parallel
Resistance and Reactance
for Induction Motor Load

1 per unit, and must be modified if the system base is different.

No claim is made as to the accuracy of results obtained with the equivalent impedances of figure 16 when used with systems other than the one specified for this problem. Additional data would be necessary before a general statement could be made. However, for this particular system, the method can be extended to cover faults other than the simple three phase fault.

For a line-to-line fault the positive-sequence and negative-sequence equivalent circuits are required for a solution, and for single-line-to-ground and double-line-to-ground faults, the zero-sequence circuit is required in addition. Usually, in small-size induction motors, the neutral is not grounded, and the induction motor will not be a part of the zero-sequence equivalent circuit. In the negative-sequence circuit of the induction motor, the equivalent load resistance appears as

$$R_4 = \frac{R_r}{2 - s}, \tag{16}$$

and the effects of small decelerations or changes of slip tend to be suppressed. Therefore, a fixed value can be used to represent the negative-sequence impedance for all values of induction motor inertia constant. Corresponding to the circuit in figure 16, with an average slip of 5 per cent, these constant negative-sequence values are about R = 35 to 40 per cent and X = 20 per cent where the motor base impedance is 100 per cent. The values to be used in the positive-sequence circuit can be taken directly from figure 16.

SYNCHRONOUS MOTOR LOADS

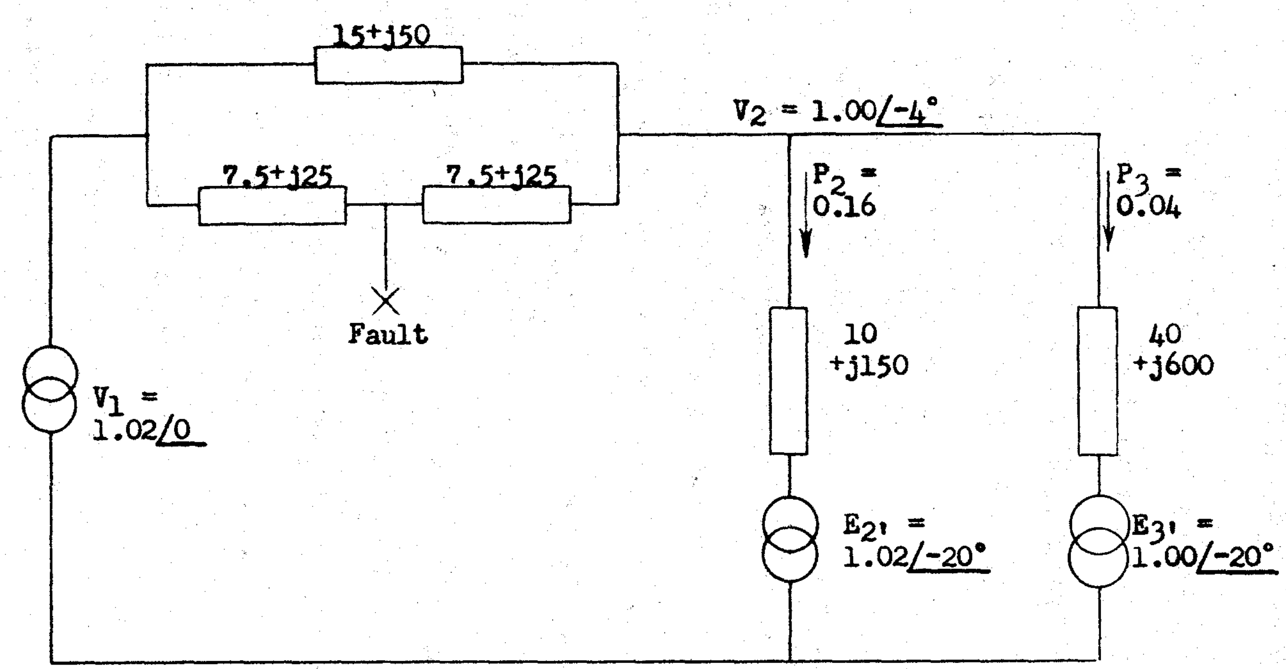
Assumptions and Procedures

The studies of the effects of synchronous motor loads were quite similar to those for the induction motor loads in that the same power system equivalent circuit and the same procedures were used. The equivalent circuit is shown in figure 17 with the induction motor replaced by the equivalent synchronous motor.

Studies were made with various combinations of inertia and power factor for motor no. 3 and the values for the swing curves for motor no. 2 were calculated for each combination. The exact curve for each combination was found by the conventional three-machine step-by-step procedure. Then a fixed impedance was substituted for motor no. 3 and the approximate curve was found by the two-machine procedure described in the preceding section. The rating of motor no. 3 was one-fourth that of no. 2. Studies were attempted for conditions where the two motors were of equal rating, but it soon became apparent that it would be impossible to substitute a fixed impedance for the synchronous motor no. 3.

The assumptions made were as follows:

1. The sending end of the transmission line is connected to an infinite bus, representing a system which is very large with respect to the portion under study.
2. The receiving end load consists of two synchronous motors--one having a rating of one-fourth the other, and both loaded to 80 per cent of



All impedances are percentage values.
Pre-fault values of power and voltage are shown.

Synch. Motor No. 2 | **Synch. Motor No. 3**

Figure 17

Network Analyzer Power System Equivalent Circuit for Synchronous Motor Studies

their ratings to be consistant with the previous induction motor studies.

3. Since the time constant of a synchronous motor's field circuit is much longer than the transient interval under consideration, the flux linkages of the field winding may be assumed constant during the interval. Therefore, the synchronous motors' voltages back of their transient reactances are also assumed to be constant at levels determined by the initial pre-fault conditions.
4. The asynchronous torques of the motor damper windings are neglected as they are usually small in comparison to the synchronous torques.
5. The inertia constant for motor no. 2 is 2.0 joules per volt-ampere, and for no. 3 it is 0.5, 2.0, and 8.0 for each of the three groups of studies. Two joules per volt-ampere is an average value of inertia constant for synchronous motors and their connected loads.
6. A three-phase fault occurs at the middle of one of the transmission lines and is cleared 0.2 second later by simultaneous opening of circuit breakers at both ends of the faulted line.
7. The time interval used in the step-by-step calculations is 0.05 second.

Nine studies were made with different combinations of inertia and power factors for machine no. 3. The power factors used were unity, 0.8 lag and 0.8 lead, and the inertia constants were 0.5, 2.0, and 8.0 joules per volt-ampere. The first six studies with inertia constants of 0.5 and 2.0 were made on the network analyzer. The last three, for the inertia constant of 8.0, were computed long-hand as the analyzer was not available at that time. In all cases, the initial pre-fault conditions of $V_2 = 1.00$, $P_2 = 0.16$, $P_3 = 0.04$, and the desired power factors were obtained by adjustment of the magnitude and angle of V_1, E_2' and E_3'.

The fault was then placed on the transmission line and the new values of P_2' and P_3' were measured. P_2' and P_3' were the power outputs of the two synchronous motors, and the differences between these values and the pre-fault power outputs gave the accelerating powers. The swing angles for the two motors were then found by the conventional step-by-step process using the following equations:

for motor no. 2 for all cases

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 67.5\, P_{a(n-1)}, \qquad (17)$$

for motor no. 3 with H = 0.5

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 1080\, P_{a(n-1)}, \qquad (18)$$

for motor no. 3 with H = 2.0

$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 270\, P_{a(n-1)}, \qquad (19)$$

and for motor no. 3 with H = 8.0

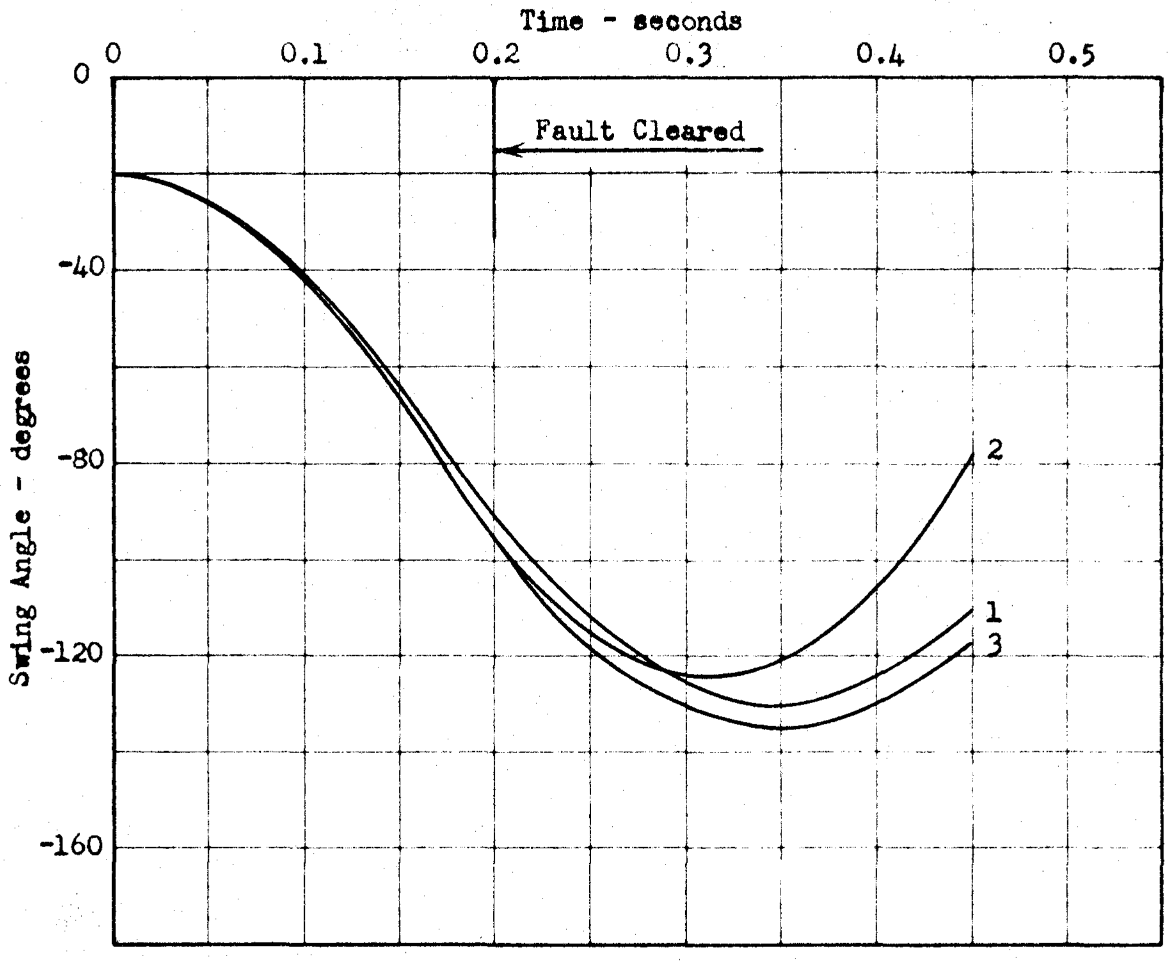
$$\delta_n = \delta_{(n-1)} + \Delta\delta_{(n-1)} + 67.5\, P_{a(n-1)}. \qquad (20)$$

For the first study where motor no. 3 operated at unity power factor and had an H = 0.5, the calculations are summarized in table 7 and the resulting exact curve is shown in figure 18. The calculations for the remaining eight exact curves were similar and the curves themselves are shown in figures 19-26 inclusive.

Table 7

Step-by-step Calculations for
Synchronous Motor Swing Curve
with Parallel Synchronous Motor Load

Time	$P_{2'}$	P_a	$67.5P_a$	$\Delta\delta_{2'}$	$\delta_{2'}$	$P_{3'}$	P_a	$1080P_a$	$\Delta\delta_{3'}$	$\delta_{3'}$
0.00-	+0.155				- 20.0	+0.036				- 20.0
0.00+	-0.005				- 20.0	0.0				- 20.0
0.00A	+0.075	-0.080	- 5.4		- 20.0	+0.018	-0.018	- 19.5		- 20.0
				- 5.4					- 19.5	
0.05	+0.005	-0.150	-10.1		- 25.4	+0.017	-0.019	- 20.6		- 39.5
				-15.5					- 40.1	
0.10	+0.045	-0.110	- 7.4		- 40.9	+0.032	-0.004	- 4.3		- 79.6
				-22.9					- 44.4	
0.15	+0.088	-0.067	- 4.5		- 63.8	+0.043	+0.007	+ 7.6		-124.0
				-27.4					- 36.8	
0.20-	+0.110				- 91.2	+0.023				-160.8
0.20+	+0.350				- 91.2	+0.067				-160.8
0.20A	+0.230	+0.075	+ 5.1		- 91.2	+0.045	+0.009	+ 9.7		-160.8
				-22.3					- 27.1	
0.25	+0.305	+0.150	+10.1		-113.5	+0.014	-0.022	- 23.8		-187.9
				-12.2					- 50.9	
0.30	+0.265	+0.110	+ 7.4		-125.7	-0.064	-0.100	-108.0		-238.8
				- 4.8					-158.9	
0.35	+0.315	+0.160	+10.8		-130.5	+0.020	-0.016	- 17.3		- 37.7
				+ 6.0					-176.2	
0.40	+0.265	+0.110	+ 7.4		-124.5	-0.029				-213.9
				+13.4						
0.45					-111.1					

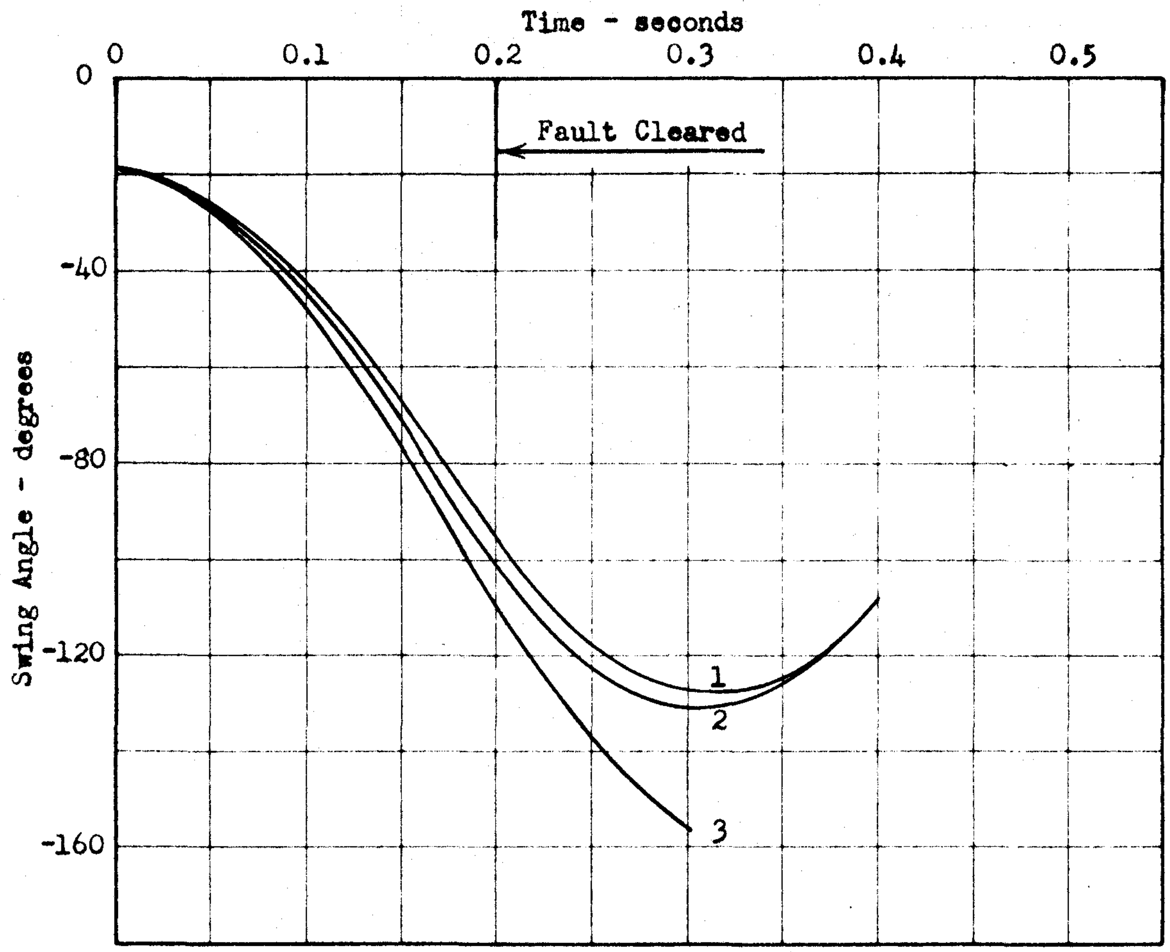


1 - Exact swing curve for synchronous motor no. 2.
2 - Curve for no. 2 when no. 3 is replaced by a constant resistance equal to 10 per cent of its pre-fault value.
3 - Curve for no. 2 when no. 3 is replaced by a constant resistance equal to 5 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=1.00.
Synch. motor no.3: G_3=0.25; H_3=0.5; load=80%; pf=1.00.

Figure 18

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

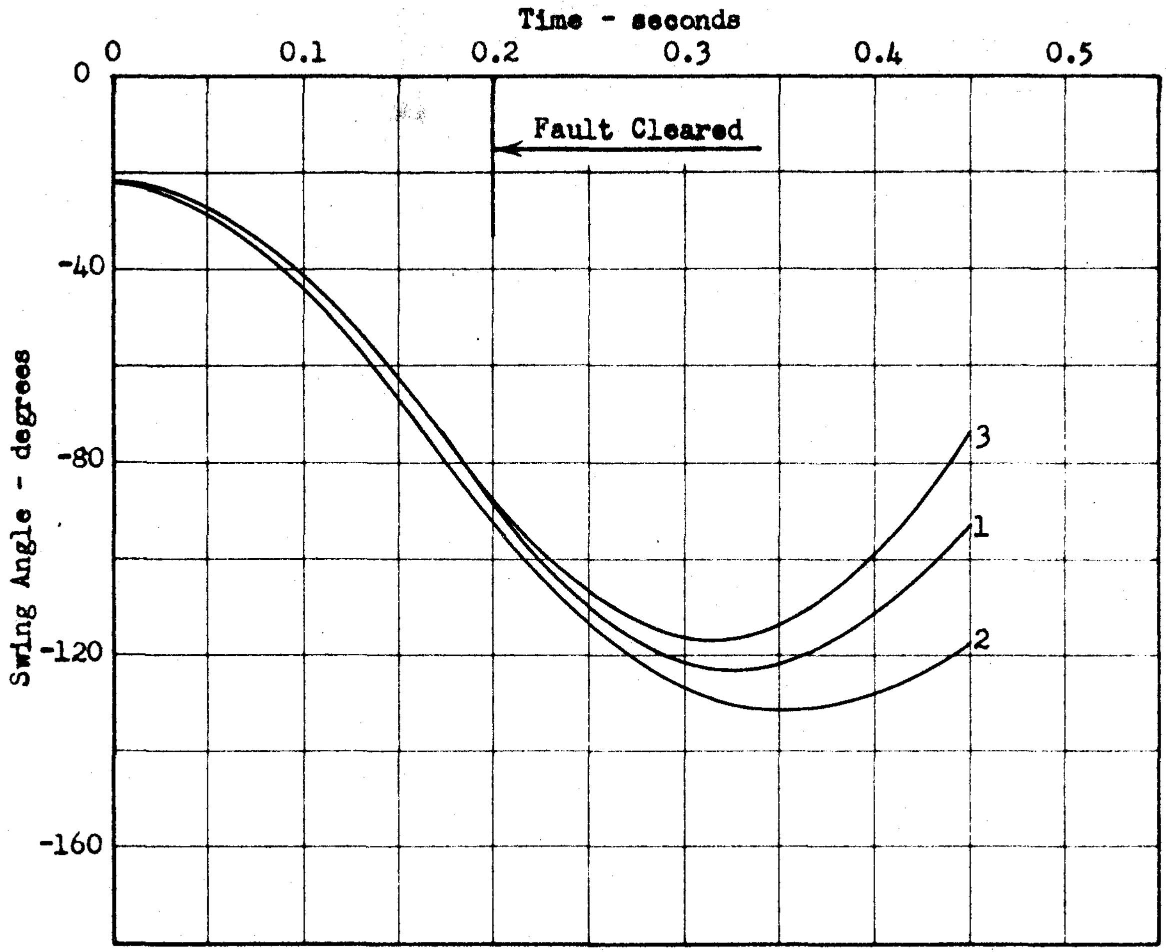


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 10 per cent of its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 5 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 leading.
Synch. motor no.3: G_3=0.25; H_3=0.5; load=80%; pf=0.8 lagging.

Figure 19

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

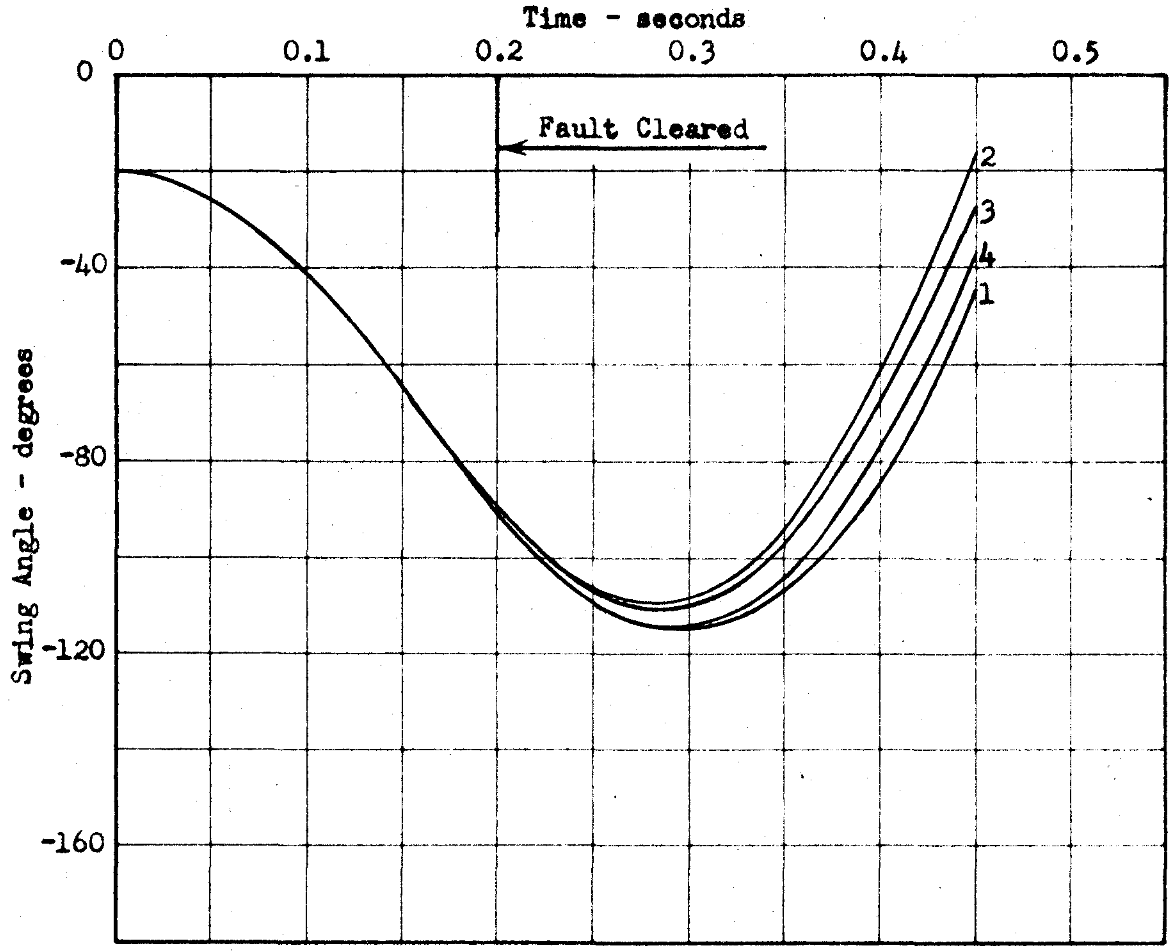


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 10 per cent of its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 lagging.
Synch. motor no.3: G_3=0.25; H_3=0.5; load=80%; pf=0.8 leading.

Figure 20

Swing Curves for Synchronous Motor
with Parallel Synchronous Motor Load

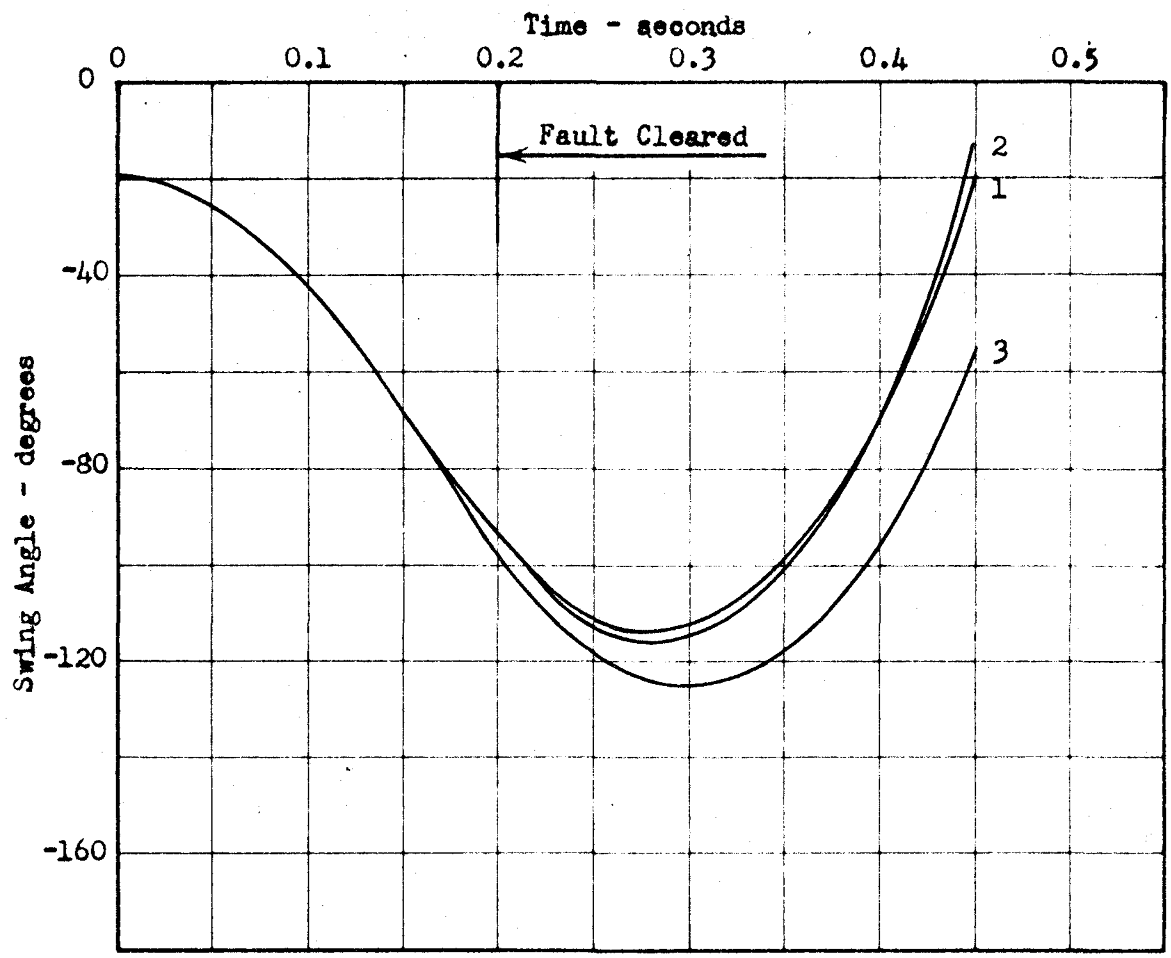


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is removed completely.
3 - Curve for no.2 when no.3 is replaced by its pre-fault resistance.
4 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=1.00.
Synch. motor no.3: G_3=0.25; H_3=2.0; load=80%; pf=1.00.

Figure 21

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

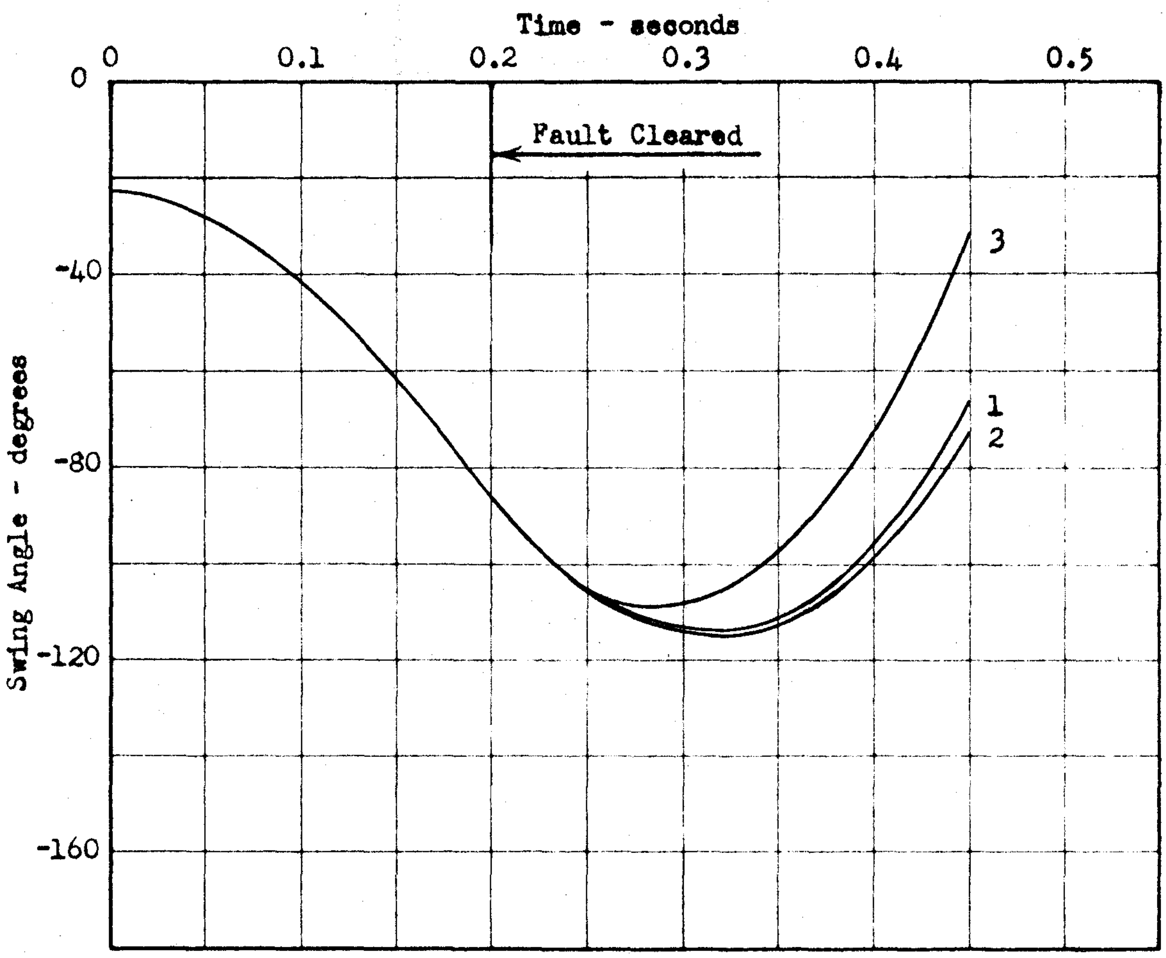


1 - Exact swing curve for synchronous motor no.2.
2 -Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant impedance equal to 20 per cent of its pre-fault impedance.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 leading.
Synch. motor no.3: G_3=0.25; H_3=2.0; load=80%; pf=0.8 lagging.

Figure 22

Swing Curves for Synchronous Motor
with Parallel Synchronous Motor Load

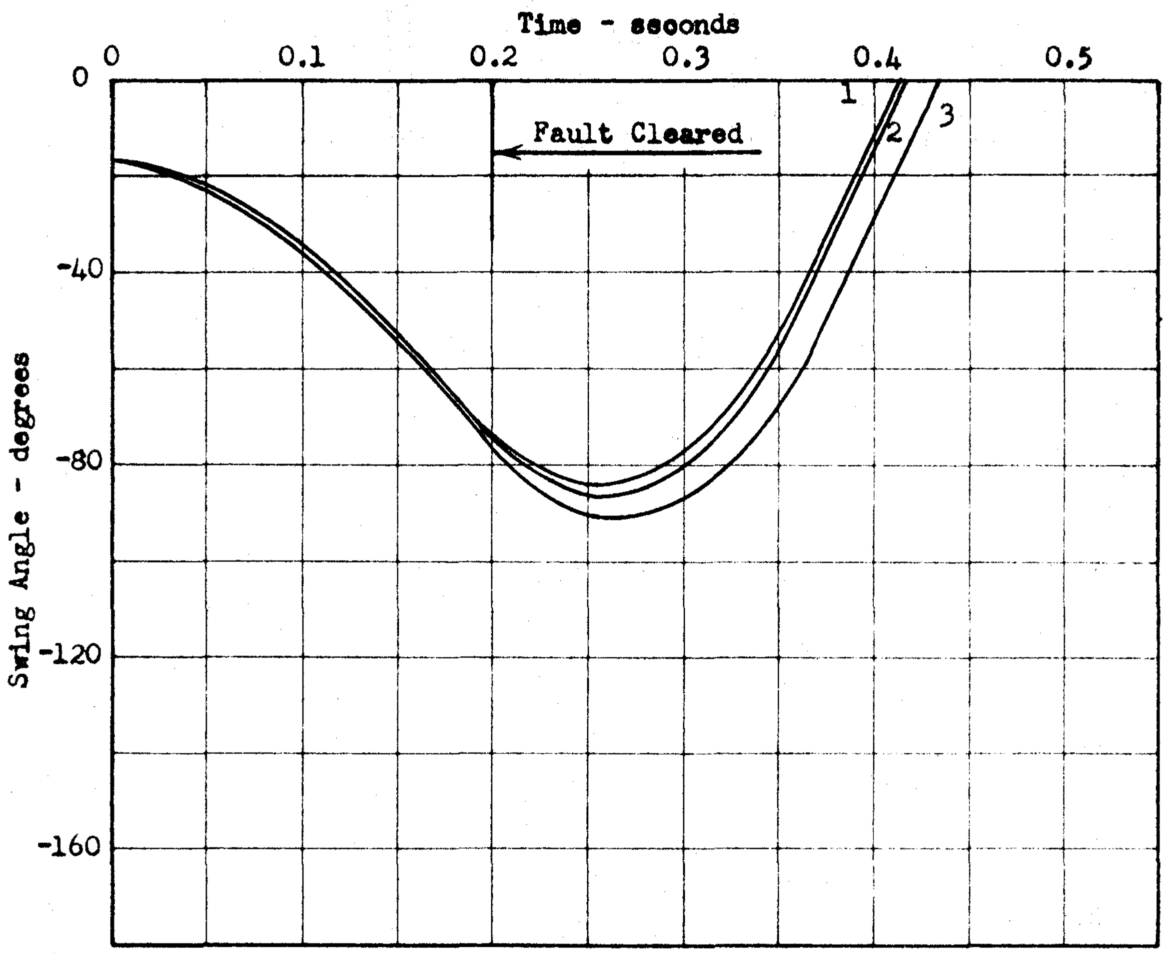


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault resistance in parallel with a constant reactance equal to 12.5 per cent of its pre-fault capacitive reactance.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 lagging.
Synch. motor no.3: G_3=0.25; H_3=2.0; load=80%; pf=0.8 leading.

Figure 23

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

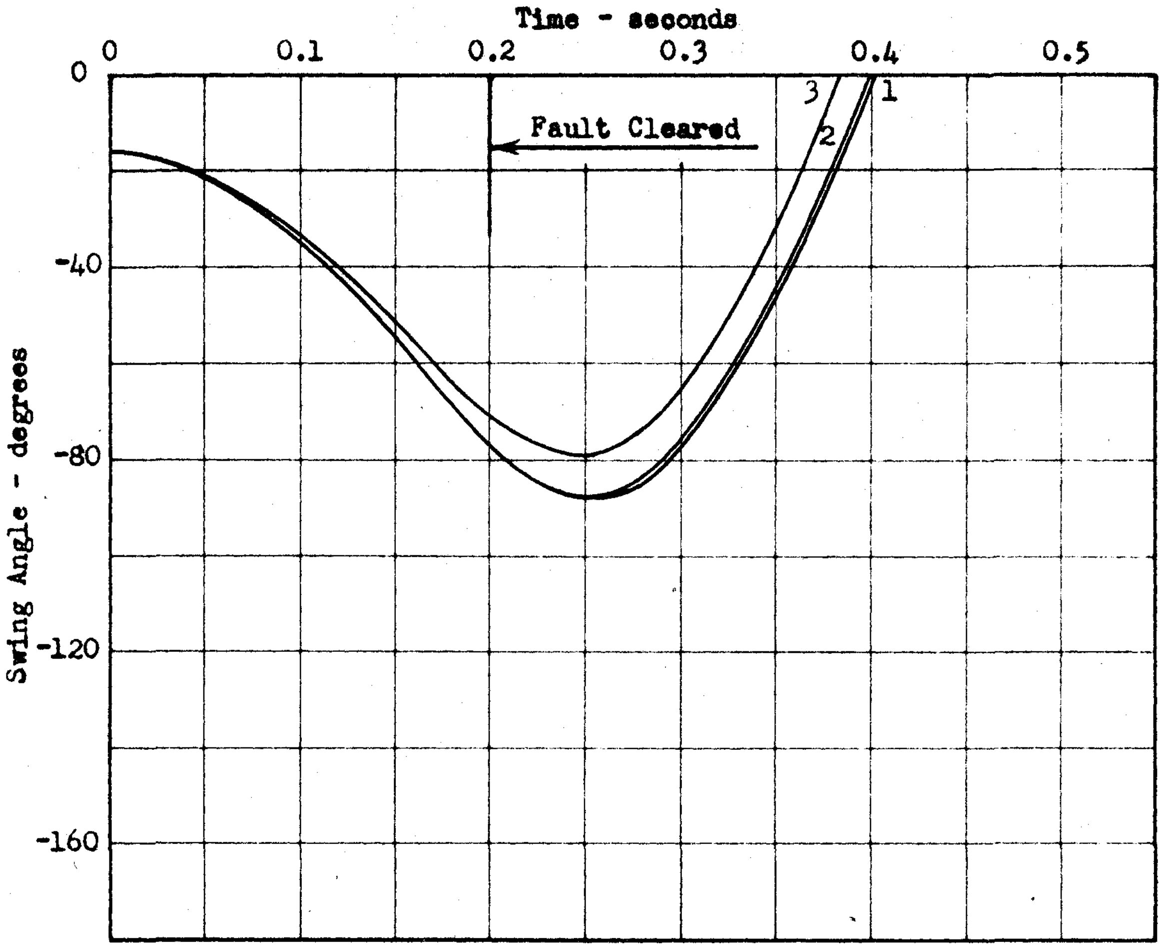


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=1.00.
Synch. motor no.3: G_3=0.25; H_3=8.0; load=80%; pf=1.00.

Figure 24

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

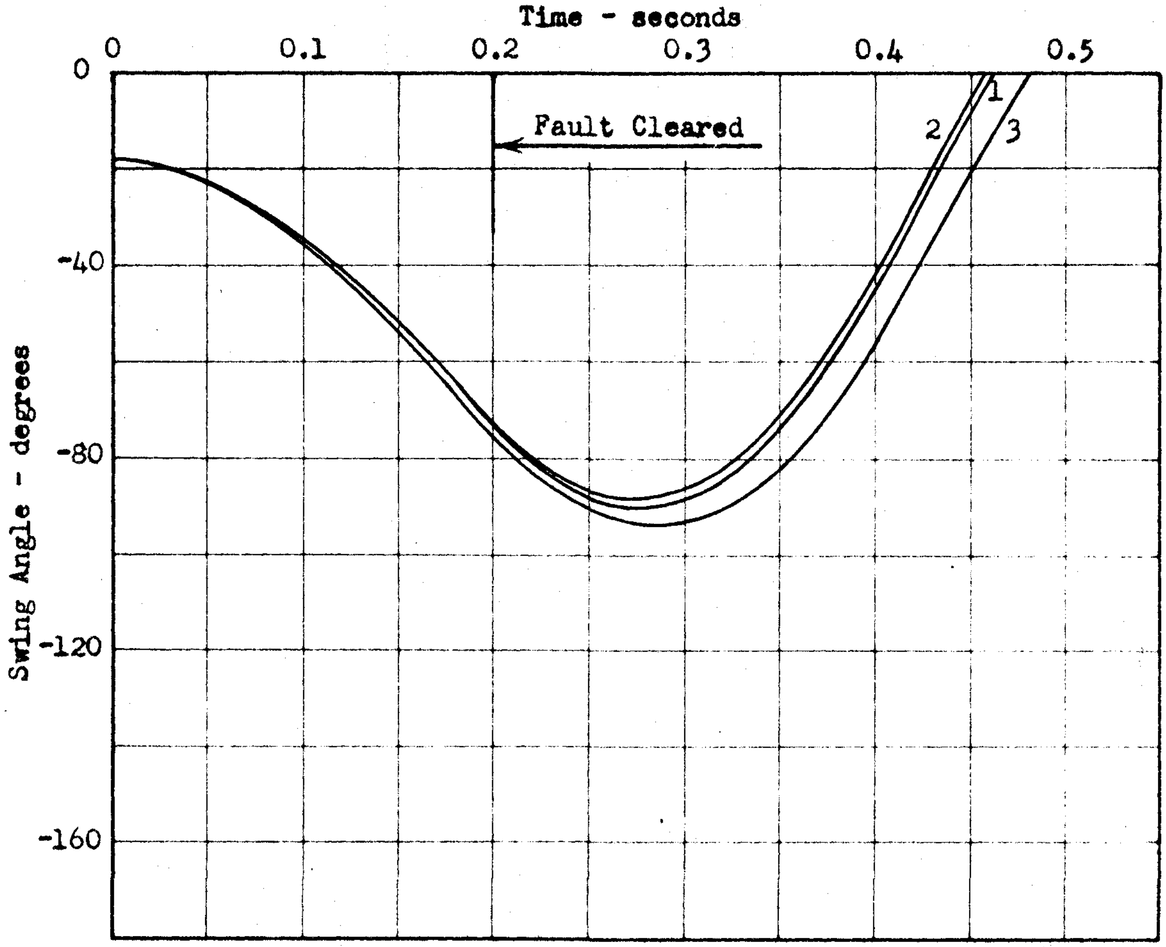


1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant impedance equal to its pre-fault impedance.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 leading.
Synch. motor no.3: G_3=0.25; H_3=8.0; load=80%; pf=0.8 lagging.

Figure 25

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load



1 - Exact swing curve for synchronous motor no.2.
2 - Curve for no.2 when no.3 is replaced by a constant resistance equal to its pre-fault value.
3 - Curve for no.2 when no.3 is replaced by a constant resistance equal to 20 per cent of its pre-fault value.

Synch. motor no.2: G_2=1.00; H_2=2.0; load=80%; pf=0.8 lagging.
Synch. motor no.3: G_3=0.25; H_3=8.0; load=80%; pf=0.8 leading.

Figure 26

Swing Curves for Synchronous Motor with Parallel Synchronous Motor Load

Each of the nine studies was repeated with two or more fixed impedances replacing the synchronous motor no. 3. The calculations are similar to those summarized in table 5 and the results are presented in figures 18-26 along with the corresponding exact curves. The studies represented by figures 22 and 23 were among the first ones made, and from these it appeared that a pure resistance was more nearly equivalent to the transient effect of the synchronous motor than a parallel resistance and reactance combination, even when the pre-fault power factor was other than unity. Therefore, only a pure resistance was used for the equivalent representation in the remainder of the studies.

Results

In figure 18, the equivalent resistance is somewhere between 5 and 10 per cent of the pre-fault resistance, probably about 6 per cent. Similarily, from figure 20, the equivalent resistance appears to be about 15 per cent of the pre-fault value. The most suitable equivalent resistances for all nine studies are summarized in table 8 and are presented in the form of curves in figure 27. For convenience of use with the network analyzer the resistances of figure 27 are presented as percentage values referred to the synchronous motor's own base impedance of 100 per cent or 1.00 per unit. If the system base is different, the values of the equivalent resistances must be modified.

As with the induction motor studies, no claim can be made as to the accuracy of results obtained with these equivalent resistances when used

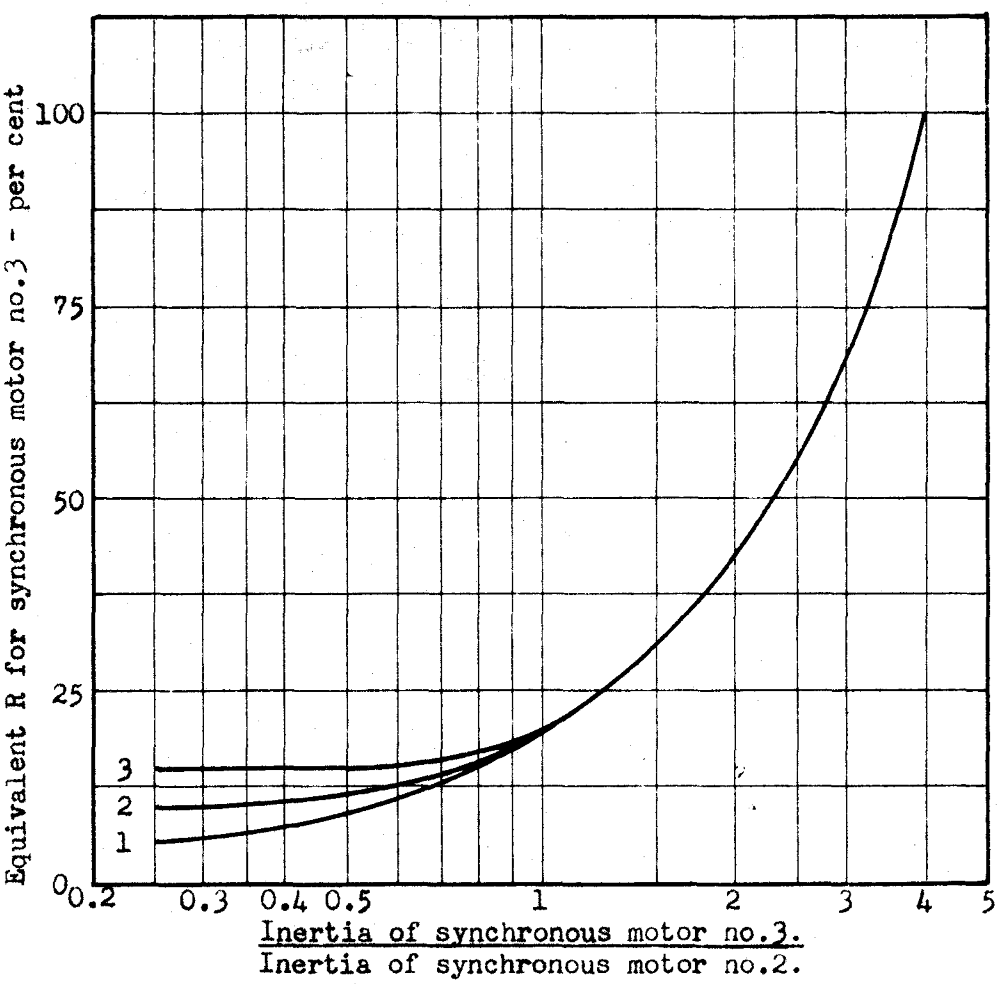
Table 8

Summary of Resistances Which Are
Equivalent to Synchronous Motor
No. 3 during Transient Period

Synchronous motor no. 2: $G_2 = 1.00$; $H_2 = 2.0$.
Synchronous motor no. 3: $G_3 = 0.25$.

	$H_3 = 0.5$	$H_3 = 2.0$	$H_3 = 8.0$
$pf_3 = 1.00$	6%	20%	100%
$pf_3 = 0.8$ lagging	10	20	100
$pf_3 = 0.8$ leading	15	20	100

These equivalent resistances are referred to the synchronous motor's own base impedance 100 per cent.



1.- Pre-fault power factor of motor no.3 = 1.00.
2 - Pre-fault power factor of motor no.3 = 0.80 lagging.
3 - Pre-fault power factor of motor no.3 = 0.80 leading.

$$\frac{\text{Rating of synchronous motor no.3}}{\text{Rating of synchronous motor no.2}} = 0.25$$

Figure 27

Synchronous Motor
Equivalent Resistance

with other than the specified system. It would be necessary to make individual studies of other types of systems before a general statement could be made.

If faults other than the three-phase type are to be considered, then the negative-sequence impedance of the synchronous motor will be involved in the equivalent circuit. The values of negative-sequence reactance and resistance are both determined largely by the synchronous motor's damper winding and are not affected by angular changes of the rotor position. Therefore, they will remain constant at their average values which are about 24 per cent for the negative-sequence reactance and about 10 per cent for the negative-sequence resistance (16, p. 166). The equivalent resistance to simulate the motor in the positive-sequence circuit may be taken directly from figure 27.

HEATING AND LIGHTING LOADS

Heating Loads

It was assumed that heating elements such as are used in ranges, space heaters and water heaters would have enough thermal inertia that their temperatures would remain constant during the transient interval. If their temperatures remain constant, then their resistances also remain constant and the pre-fault values of resistance may be used in the power system equivalent circuit during the first swing of the adjacent synchronous machine.

Incandescent Lighting Loads

Incandescent lamp filaments have a relatively low thermal inertia, and their resistances vary even with 60 cycle changes of voltage. The power versus voltage relationship is given by the following equation (1, p. 724).

$$\log P = A_3(\log V)^2 + B_3 \log V, \qquad (21)$$

where P = per unit power,

V = per unit voltage,

A_3 = 0.057 for 40 - 150 watt gas filled lamps,

0.083 for 200 watt and larger gas filled lamps,

B_3 = 1.52 for 40 - 150 watt gas filled lamps,

1.54 for 200 watt and larger gas filled lamps.

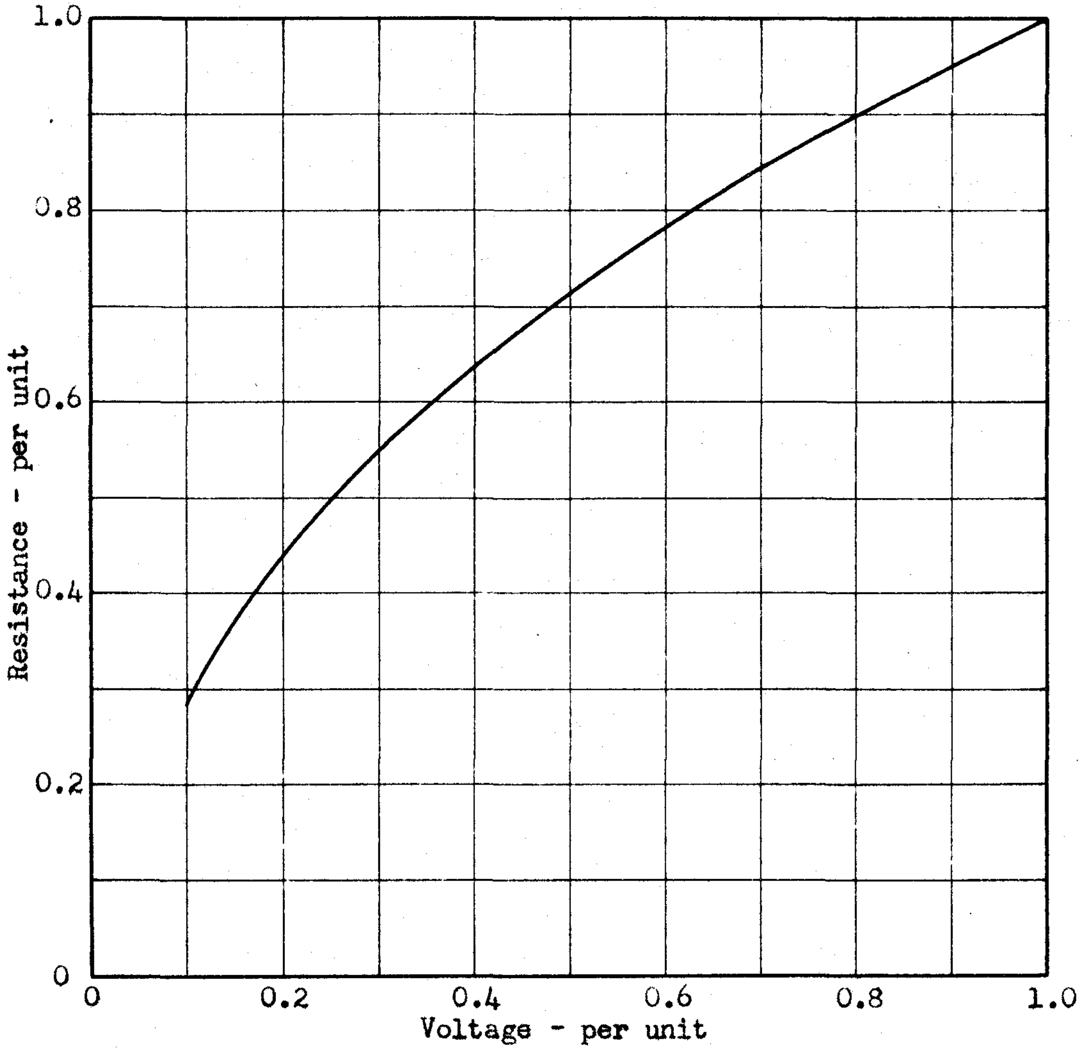
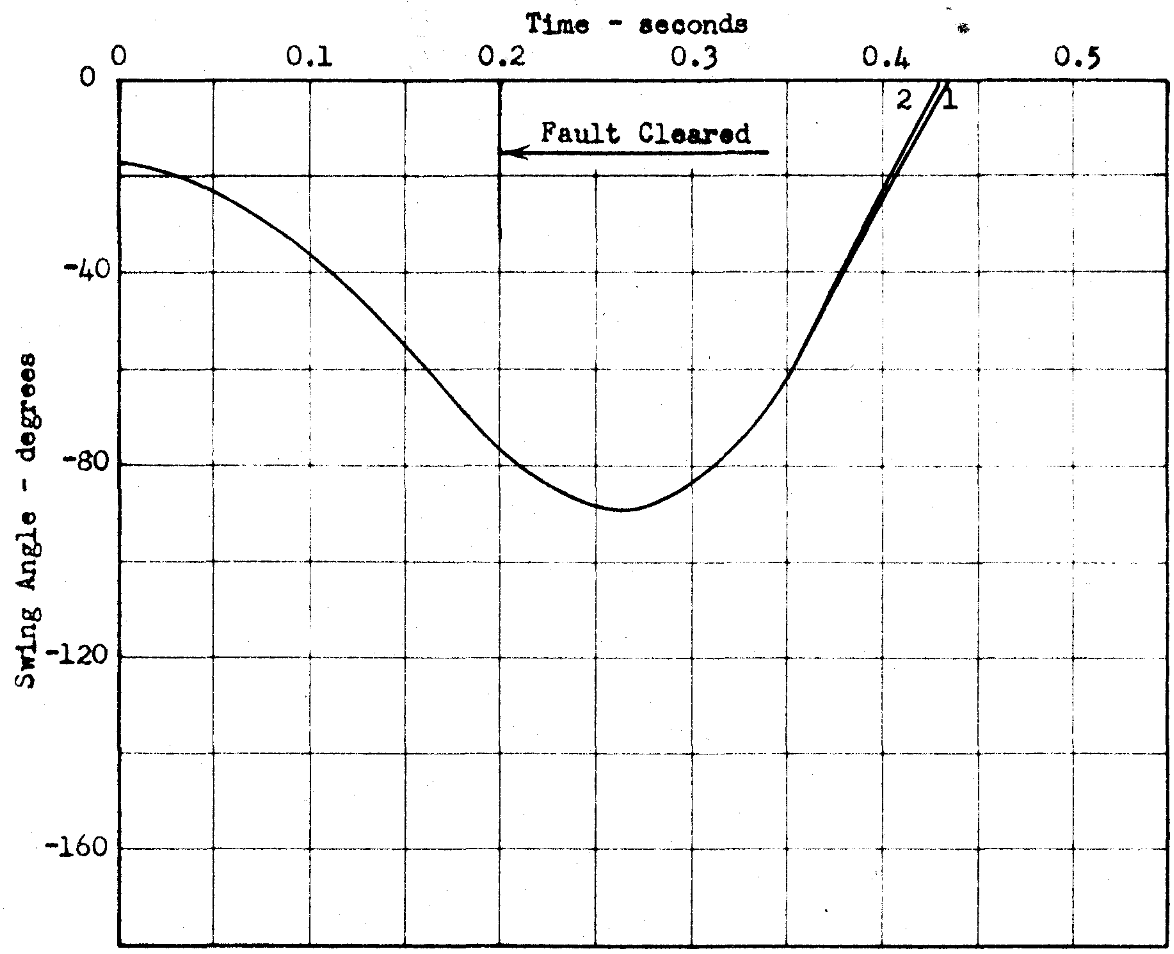


Figure 28

Resistance Characteristic
for Tungsten Filament Lamp



1 - Exact swing curve for synchronous motor.
2 - Curve with incandescent lighting load resistance held constant at its pre-fault value.

Synchronous motor: G_s=1.00; H_s=2.0; load= 80%; pf=1.00.
Incandescent lighting load: G_L=1.00; load=100%; pf=1.00.

Figure 29

Swing Curves for Synchronous Motor
with Parallel Incandescent Lighting Load

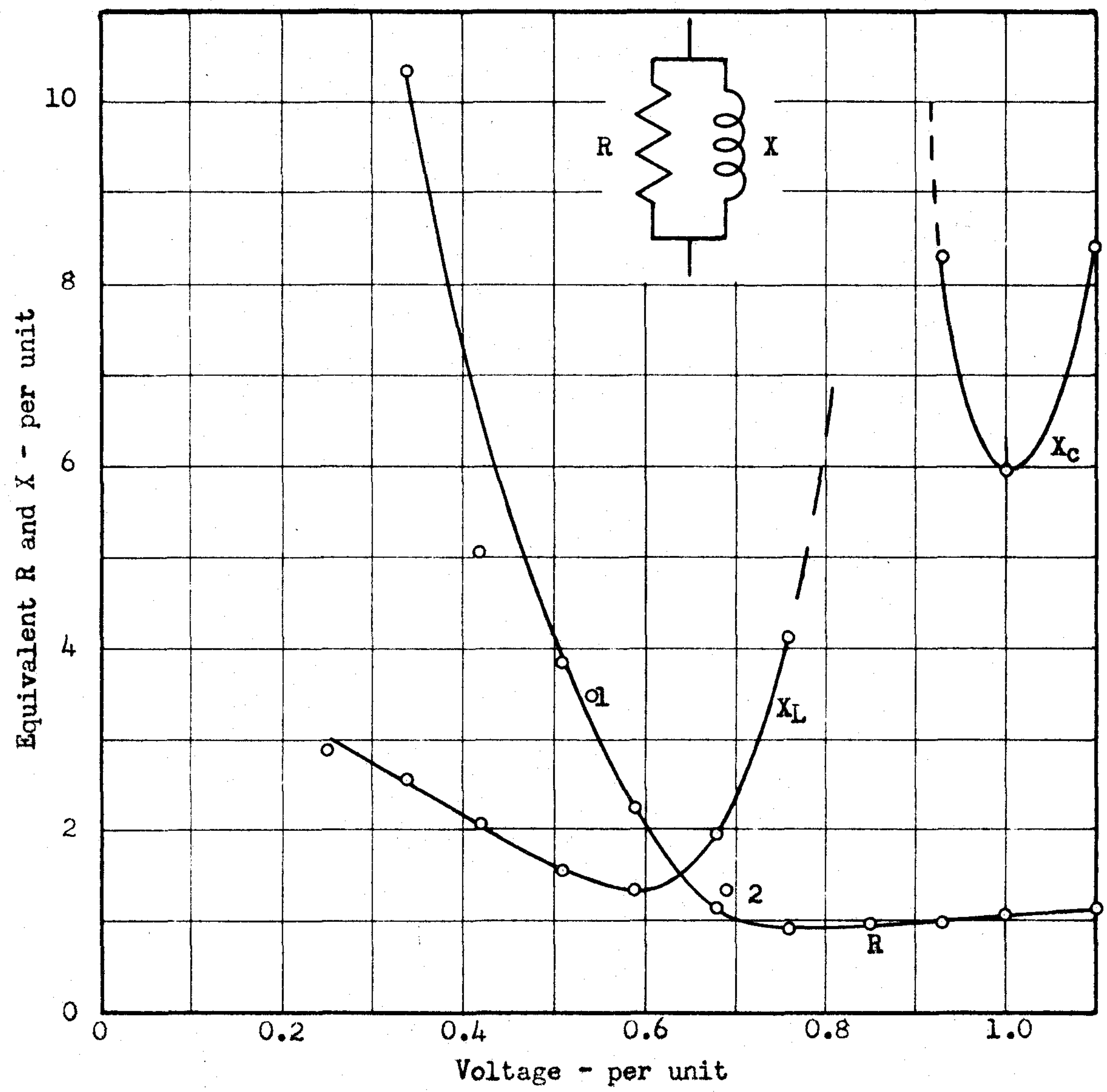
The power and corresponding resistance values were computed from this equation using average values of A_3 and B_3, and the resulting resistance versus voltage curve is shown in figure 28.

The effect of the incandescent lamp load was studied using the same power system equivalent circuit as in the preceding two sections. The lighting load was assumed to be one per unit and the adjacent synchronous motor was assumed to be operating at unity power factor and at 80 per cent of its rated load. After the fault was applied, the load voltage dropped and with the aid of figure 28 the equivalent lighting load resistance was adjusted to a new value corresponding to the decreased load voltage. This readjustment was repeated at each step in the step-by-step procedure. The results of these step-by-step calculations are presented as the exact curve in figure 29. Figure 29 also shows the swing curve which is obtained if the lighting load equivalent resistance is held constant at its pre-fault value. As the two curves are practically coincident, it may be concluded that for this particular problem, the incandescent lighting load resistance may be held constant at its pre-fault level throughout the transient interval.

Slimline Fluorescent Lighting Loads

In order to study the effect of a slimline fluorescent lighting load, the average characteristics of 18 slimline fixtures were obtained. Each fixture consisted of two 96T12 slimline fluorescent lamps and one General Electric series ballast no. 89G396. The characteristics, presented in figure 30, were calculated from the steady-state voltage, active power, and reactive power measurements. The transient characteristics, during the time intervals of this problem, were identical to the steady-state characteristics, as evidenced by the oscillogram copy in figure 31. This oscillogram is a recording of the transient performance of all 18 fixtures and it shows that the power reaches a steady value within one cycle after an abrupt voltage change. The voltage drop corresponds to the instant of application of a fault to the hypothetical power system, and the level of voltage was made to be about equal to the average of the load voltages during the fault interval in the previous studies. The voltage rise occurs 7.2 cycles or 0.12 second later and corresponds to the removal of the fault. Again the voltage level is about the average of the post-fault voltages of the preceding studies. The relative levels of voltage, current, and power during and after the fault correspond closely to the measured steady-state values.

The procedure for the transient study of the slimline fluorescent load was exactly the same as that for the incandescent load. The same power system equivalent circuit was used except that the values for the slimline fluorescent load impedance were taken from figure 30 at each step in the step-by-step process. The result is shown in figure 32. No constant impedance could be found which would simulate this load during the transient interval. However, it was found that if the equivalent impedance was assumed to be infinite during the fault interval and to be equal to the pre-fault impedance after the fault was cleared, then a reasonable



Points 1 and 2 are resistances calculated from transient tests on 18 slimline fluorescent lamp fixtures.

Each fixture consists of two 96T12 slimline fluorescent lamps and one General Electric no. 89G396 series ballast.

Figure 30

Average Steady-state Characteristics for Slimline Fluorescent Lighting Load

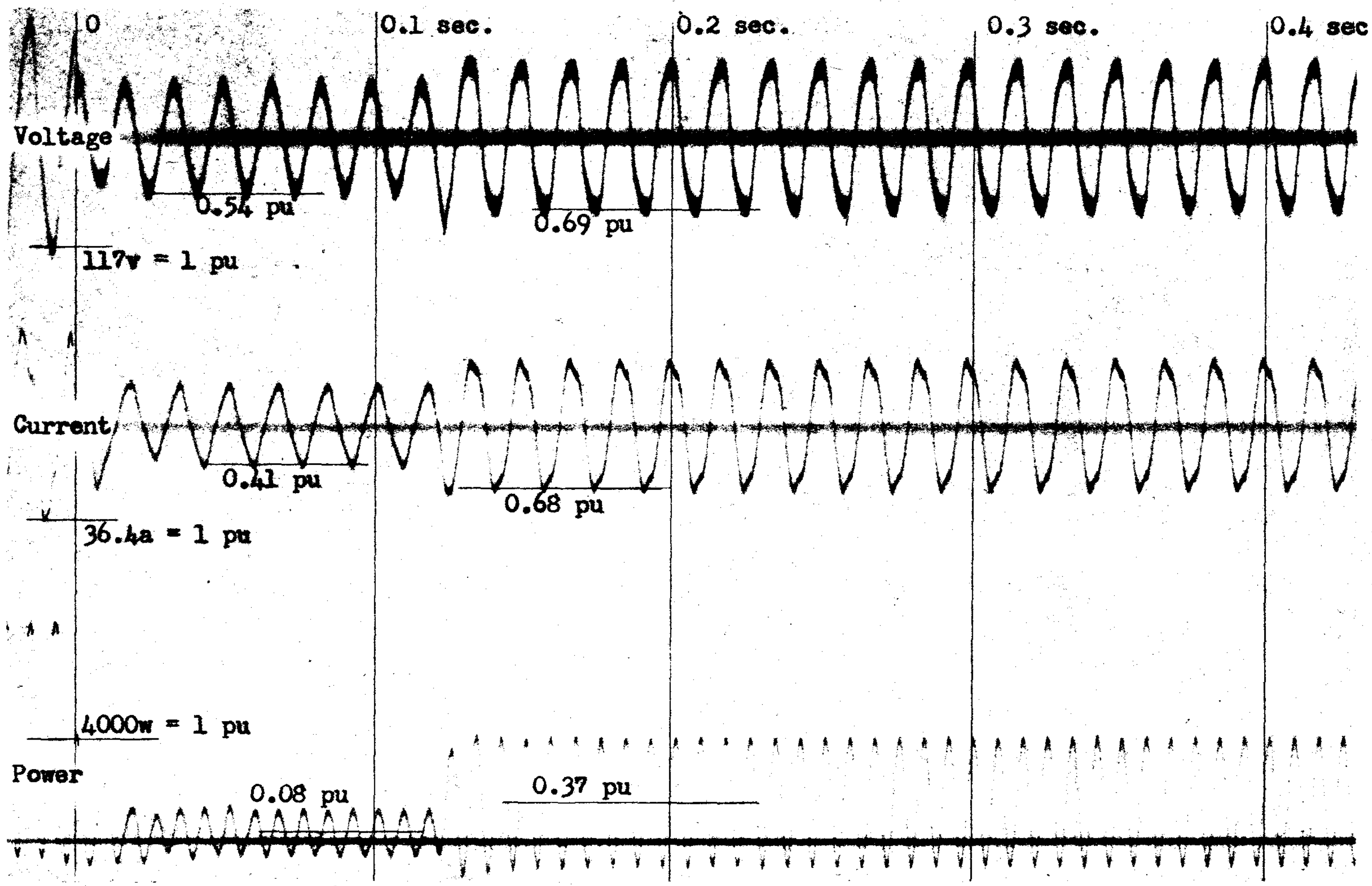
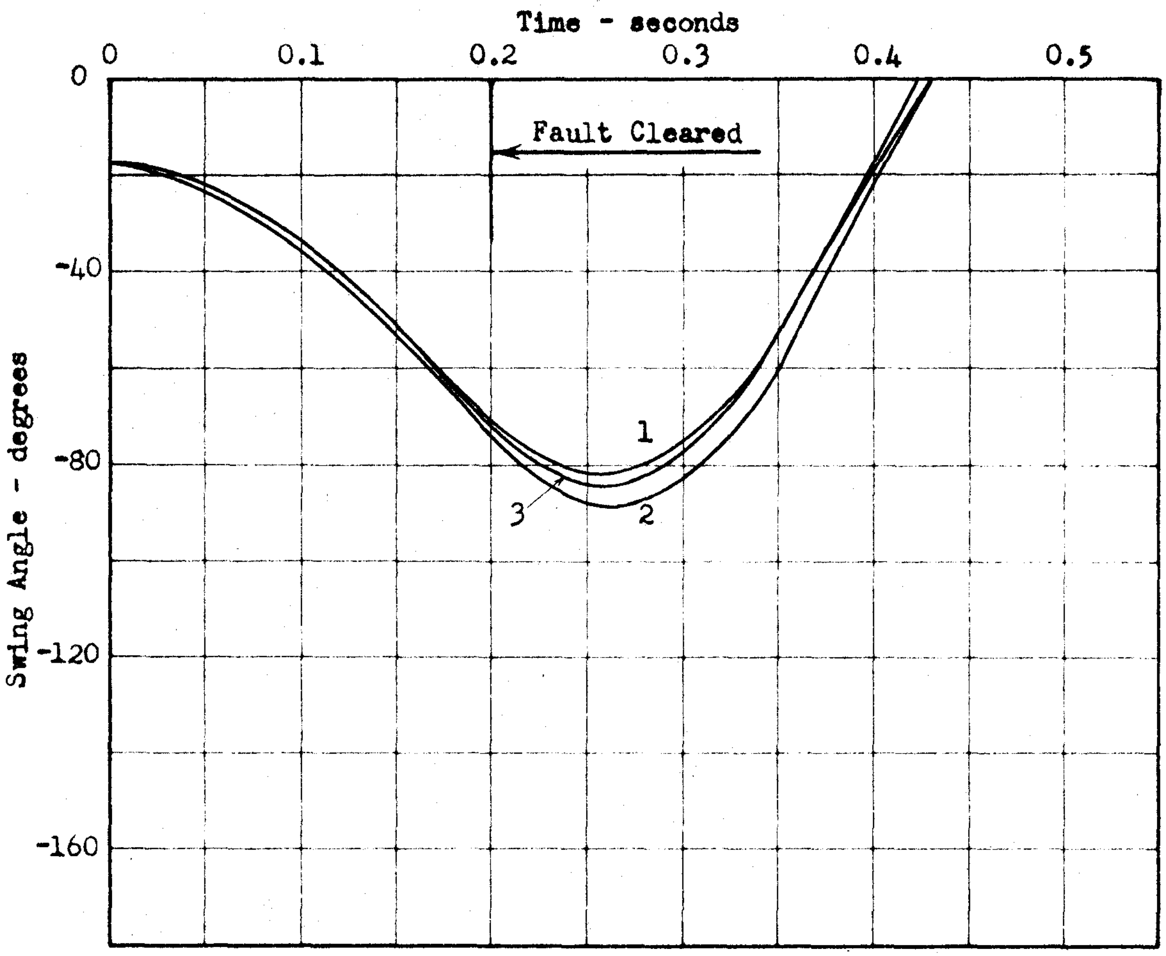


Figure 31

Transient Performance of Slimline
Fluorescent Lighting Load



1 - Exact swing curve for synchronous motor.
2 - Curve with slimline fluorescent lighting load impedance held constant at its pre-fault value.
3 - Curve with slimline lighting load represented by an infinite impedance during the fault and by its pre-fault impedance after the fault has been cleared.

Synchronous motor: G_s=1.00; H_s=2.0; load=80%; pf=1.00.
Slimline fluorescent load: G_L=1.00; load=100%; pf=0.99 leading.

Figure 32

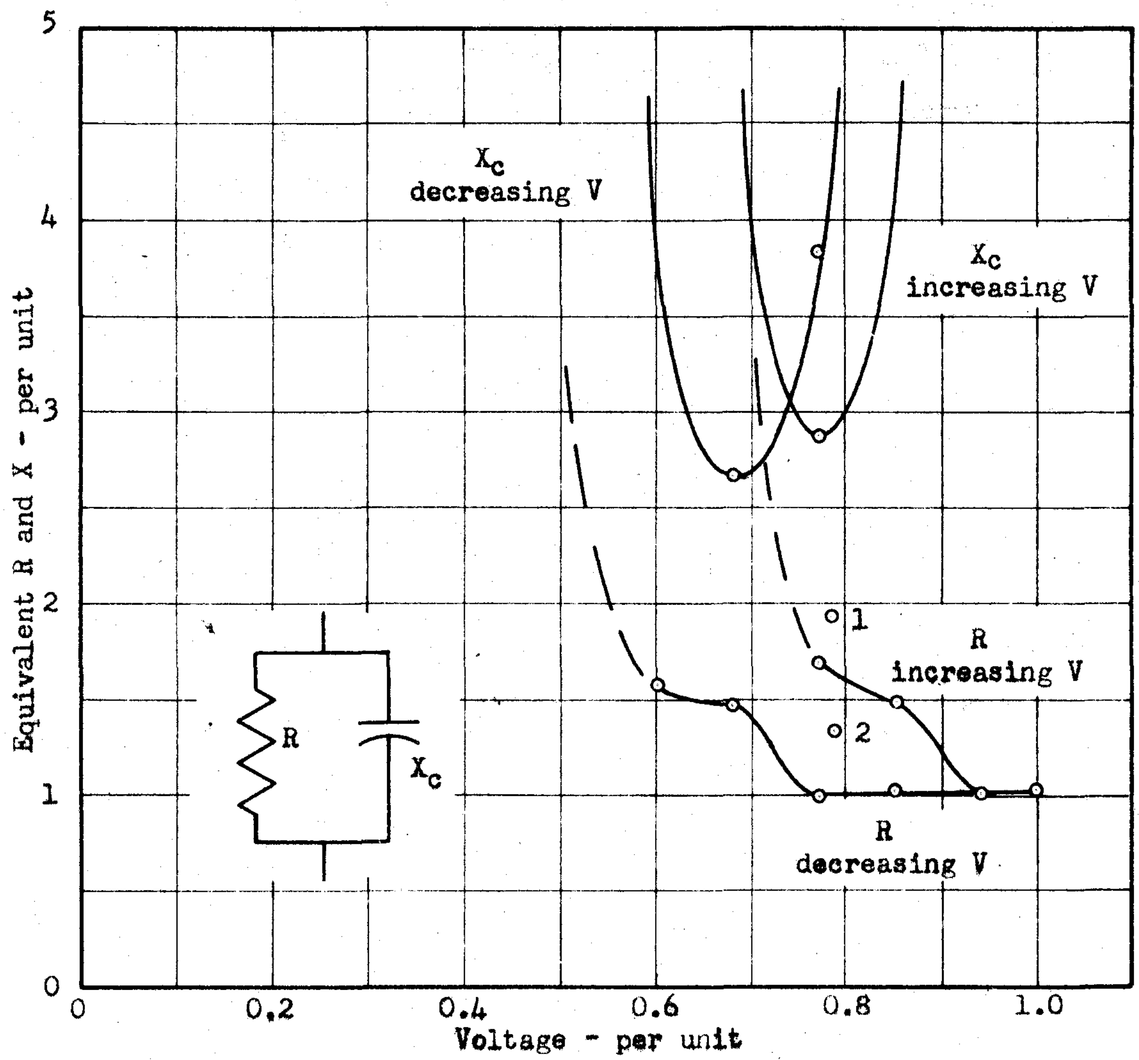
Swing Curves for Synchronous Motor with Parallel Slimline Fluorescent Lighting Load approximation to the exact curve could be obtained, as shown in figure 32. The pre-fault impedance used consisted of a resistance of 1.06 per unit in parallel with a capacitive reactance of 5.97 per unit, referred to a load base impedance of 1.00 per unit.

Starter-type Fluorescent Lighting Loads

Characteristics for a starter-type fluorescent lighting load were obtained from a group of six fixtures, each with two 40-watt lamps. Two of the fixtures were equipped with Sylvania no. G-1056J two-lamp ballasts and the remaining four with Jefferson Electric Co. no. 234-881-012 two-lamp ballasts. The characteristics were calculated from measured values of voltage, active power, and reactive power, and are presented in figure 33 as a parallel combination of resistance and reactance and in figure 34 as a series combination.

The transient performance of the starter-type fluorescent lamp load is shown in the oscillogram of figure 35. The voltage levels were again made to be the averages of the fault and post-fault load voltages of the previous studies. About four cycles is required for the power to decrease to a steady value after the abrupt decrease in voltage in contrast to the one cycle required for the slimline lamps. When the voltage rises abruptly, the power rises to a steady value in about 1.5 cycles, stays at that level for about 4.5 cycles, and then slowly increases to the higher level.

The load resistance as calculated from the oscillogram voltage and

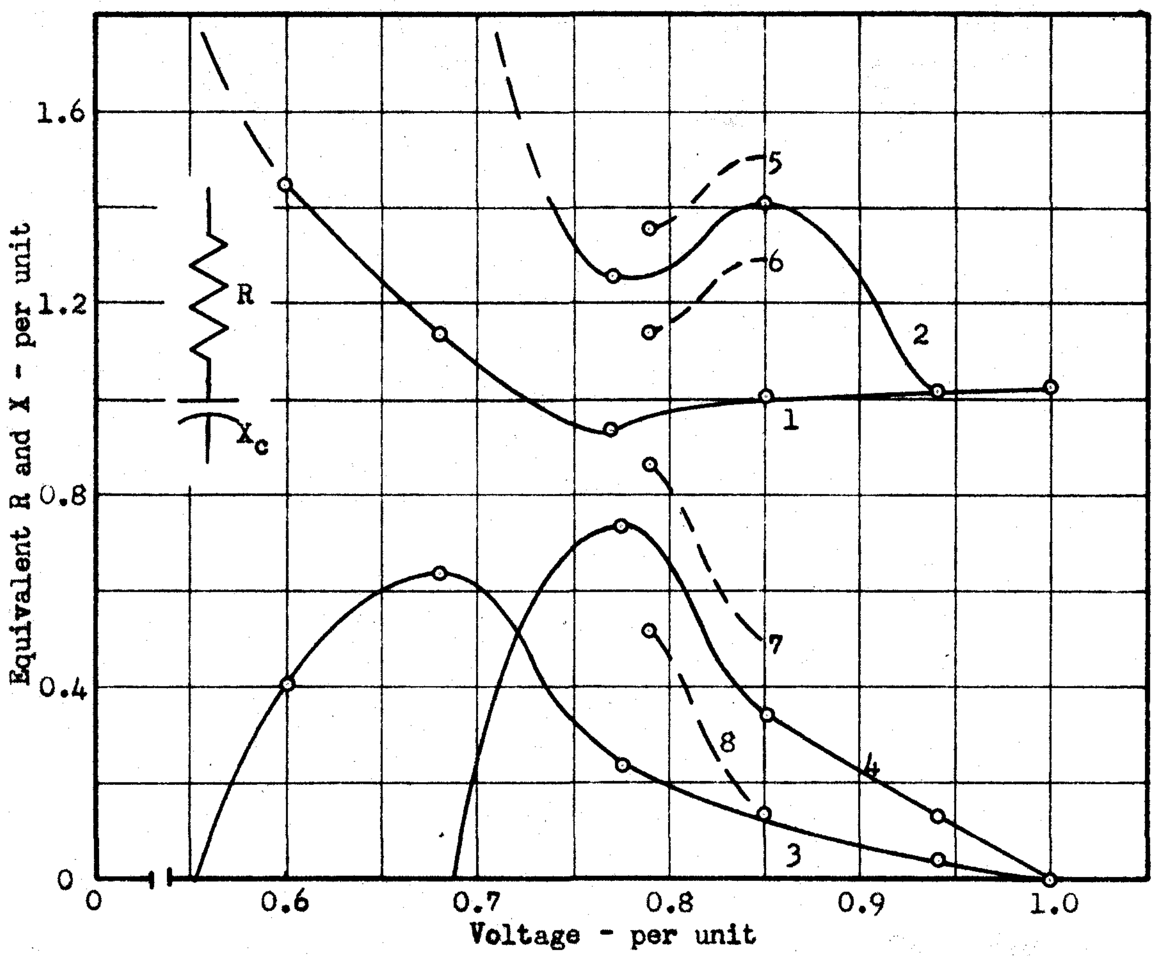


Points 1 and 2 are resistances calculated from transient tests.

Values are averages for six fixtures, each with two 40 watt lamps. Two of the fixtures have Sylvania no. G-1056J two-lamp ballasts and the remaining four have Jefferson Electric Co, no.234-881-012 two-lamp ballasts.

Figure 33

Average Steady-state Characteristics for Starter-type Fluorescent Lighting Load



1 - Steady-state resistance for decreasing voltage.
2 - Steady-state resistance for increasing voltage.
3 - Steady-state capacitive reactance for decreasing voltage.
4 - Steady-state capacitive reactance for increasing voltage.
5 - Estimated transient resistance characteristic for first 0.1 second after fault clearance.
6 - Estimated transient resistance characteristic from 0.1 to 0.4 second after fault clearance.
7 - Estimated transient reactance characteristic for first 0.1 second after fault clearance.
8 - Estimated transient reactance characteristic from 0.1 to 0.4 second after fault clearance.

Figure 34

Average Series Characteristics for Starter-type Fluorescent Lighting Load

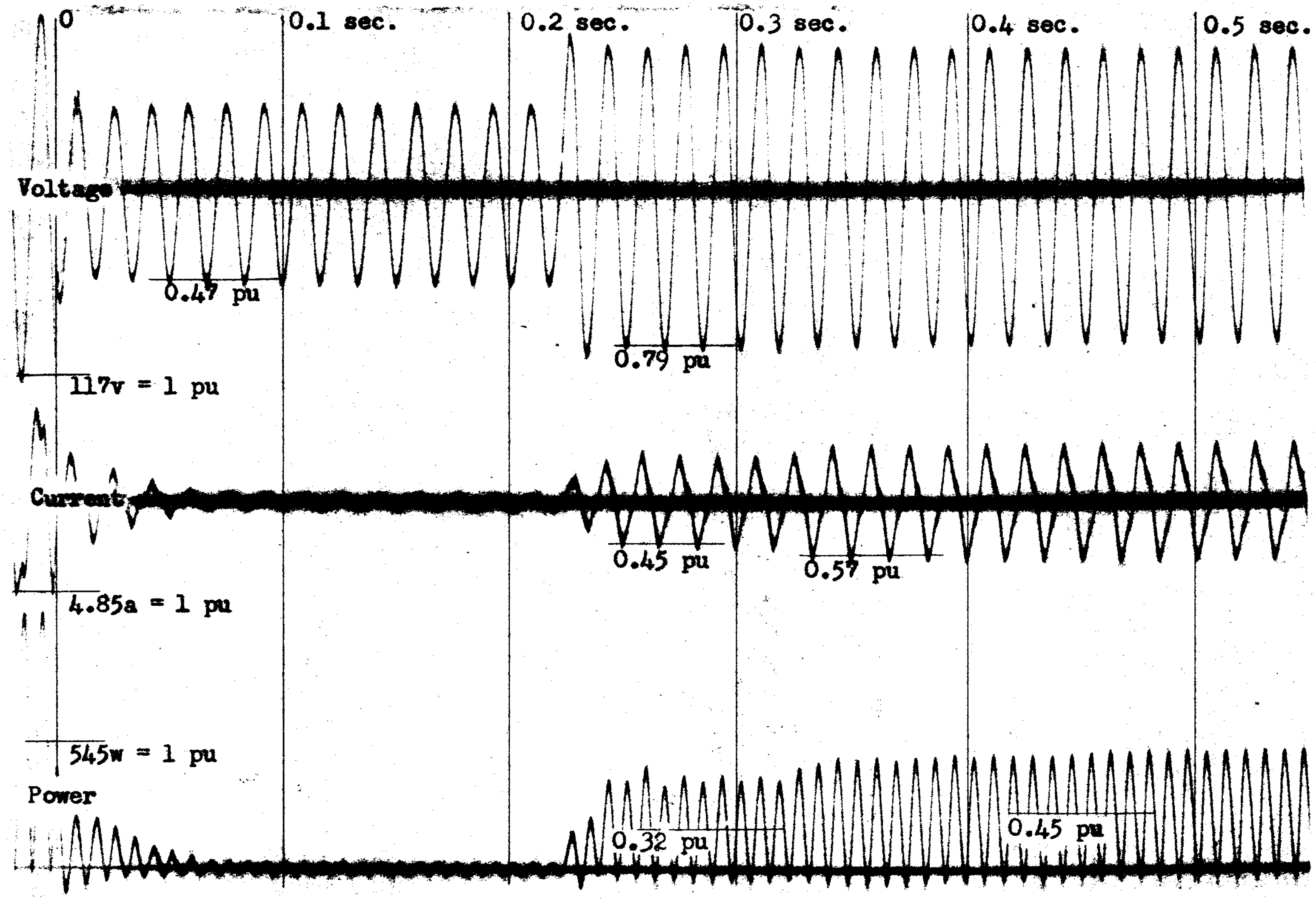


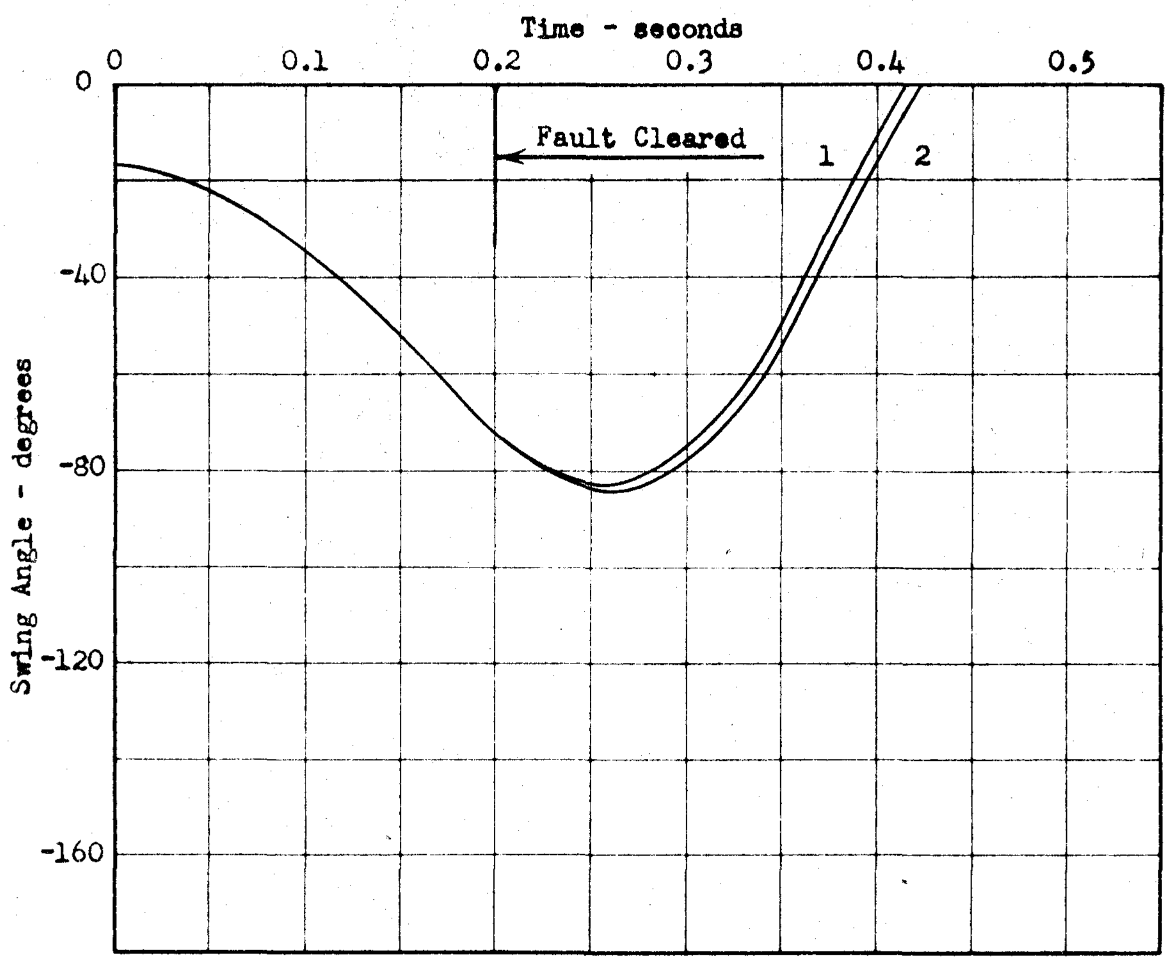
Figure 35

Transient Performance of Starter-type Fluorescent Lighting Load

power levels did not correspond to the steady-state value for that particular voltage, thus demonstrating that the transient characteristic of the starter-type fluorescent lighting load is different from the steady-state characteristic. The transient values of resistance and reactance corresponding to the two levels of recovery of power are plotted in figure 34, and estimated curves are sketched through these single points for small ranges either side of 0.79 per unit voltage.

The same power system equivalent circuit was again used to study the transient effect of this starter-type fluorescent lighting load on an adjacent synchronous motor. During the fault interval or the first 0.2 second of the step-by-step calculations, the load voltage was so low that the equivalent lighting load impedance was assumed to be infinite. From 0.2 second to 0.3 second, the load resistance and reactance corresponding to the load voltage at the particular instant were taken from the upper estimated curves of figure 34, and beyond 0.3 second, from the lower estimated curves. These step-by-step calculations resulted in the exact curve of figure 36.

As with the slimline load, no fixed impedance could be found which would produce the same transient effect as the actual load. When the equivalent impedance was assumed to be infinite during the fault interval and to be equal to the pre-fault value, thereafter, the second curve of figure 36 was obtained. The similarity of these two curves shows that pre-fault impedance may be used to represent the starter-type fluorescent light-



1 - Exact swing curve for synchronous motor.
2 - Curve with starter-type fluorescent lighting load represented by an infinite impedance during the fault and by its pre-fault impedance after the fault is cleared.

Synchronous motor: G_s=1.00; H_s=2.0; load= 80%; pf=1.00.
Fluorescent lighting load: G_L=1.00; load=100%; pf=1.00.

Figure 36

Swing Curves for Synchronous Motor with Parallel Starter-type Fluorescent Lighting Load ing load after the fault has been cleared. The pre-fault impedance used was a pure resistance of 1.00 per unit.

SUMMARY AND CONCLUSIONS

The general assumptions used in these studies are recapitulated as follows:

1. The hypothetical power system equivalent circuit of figure 7 was assumed for the induction motor load studies, and that of figure 17 for the synchronous motor studies.
2. Lighting load studies were made with a circuit similar to figure 17 but with the equivalent lighting impedance replacing synchronous motor no. 3 at the right end of the circuit.
3. The sending end of the transmission line in the assumed power system equivalent circuit is connected to an infinite bus.
4. The receiving end system consists of the particular load under study in parallel with a synchronous motor operating at 80 per cent of its 1.00 per unit rating and with an inertia constant of 2.0 joules per volt-ampere.
5. The synchronous motor voltage back of its transient reactance remains constant at its pre-fault value.
6. Damping effects are neglected.
7. A three-phase fault occurs at the middle of one of the transmission lines and is cleared 0.2 second later by the simultaneous opening of circuit breakers at both ends of the faulted line.
8. The time interval used in the step-by-step calculations is 0.05 second.

9. Under-voltage protective devices have sufficient time delay to prevent operation during the fault interval.
10. The induction motor load is the equivalent of a large number of small induction motors.
11. The decay rate of the magnetic fields and associated internal voltages in the induction motors is so rapid that it has a negligible effect on the stability of the adjacent synchronous machine.
12. The load torque on the equivalent induction motor is a constant, regardless of the slip.
13. Heating loads have sufficient thermal inertia to prevent appreciable changes in temperatures and resistances during the transient period.

Calculations based on the above assumptions led to the following conclusions:

1. When an equivalent induction motor load has an inertia constant equal to that of an adjacent synchronous motor, the induction motor may be represented by its constant pre-fault impedance throughout the transient interval.
2. If the induction motor has an inertia constant one-fourth that of the synchronous motor, it may be represented by an impedance corresponding to a 100 per cent increase in its pre-fault load torque.
3. For an inertia constant ratio of one to sixteen, the necessary load torque increase is about 230 per cent.
4. The above three conclusions are valid for cases where the equivalent

induction motor rating is equal to the synchronous motor rating and for cases where it is one-fourth of the synchronous motor rating.

5. When an equivalent synchronous motor has a rating of one-quarter that of a second adjacent synchronous motor and when their inertia constants are equal, the equivalent machine may be replaced during the transient period by a fixed resistance equal to 20 per cent of its pre-fault equivalent resistance. This is valid when the pre-fault power factor of the equivalent machine is unity, 0.8 lag, or 0.8 lead.

6. When the equivalent synchronous motor has an inertia constant of four times that of the adjacent synchronous motor, and the ratio of ratings is one to four, the equivalent machine may be replaced during the transient period by its fixed pre-fault equivalent resistance. Again, the conclusion is valid for unity, 0.8 lag, and 0.8 lead power factors.

7. If the equivalent synchronous motor's rating and inertia constant are both one-fourth of those of the adjacent machine, then for unity power factor pre-fault operation the equivalent machine may be replaced during the transient period by a fixed resistance equal to 6 per cent of its pre-fault equivalent resistance. For pre-fault power factors of 0.8 lagging and 0.8 leading, it may be replaced by 10 per cent and 15 per cent, respectively, of its pre-fault equivalent resistance.

8. Incandescent lighting loads may be represented by their pre-fault resistances throughout the transient period.

9. Neither the slimline nor the starter-type fluorescent lighting loads may be simulated by a single fixed impedance throughout the entire transient period. However, they may be represented by infinite impedances during the fault interval and by their pre-fault impedances during the post-fault period.

10. The slimline fluorescent lamp power reaches steady values in about one cycle after an abrupt voltage drop from 1.00 per unit to 0.54 per unit and after an abrupt rise back to 0.69 per unit occurring 0.12 second later. The starter-type fluorescent lamp power requires about four cycles to reach a steady value after an abrupt voltage drop from 1.00 per unit to 0.47 per unit, and about 14 cycles for an abrupt voltage rise back to 0.79 per unit occurring about 0.2 second later.

11. All of the above conclusions are valid for three-phase faults on the specified power system. For other types of faults involving the negative-sequence values, the steady-state negative-sequence impedances of the induction motor and of the synchronous motor may be used throughout the transient interval. The positive-sequence impedances are the same adjusted values as are used for the three-phase faults. For the lighting loads, the positive- and negative-sequence impedances are identical and equal to the values used for the three-phase faults.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. W. B. Boast for his encouragement, suggestions, and criticisms.

BIBLIOGRAPHY

1. Barbrow, L. E. and Meyer, J. Franklin. Characteristic Equations of Vacuum and Gas Filled Tungsten Filament Lamp. Bureau of Standards. J. of Res. 9: 721-732. 1932.

2. Byrd, H. L. and Pritchard, S. R. Solution of the Two-Machine Stability Problem. Gen. Elect. Rev. 36: 81-93. 1933.

3. Comby, J. M. and others. Emergency Control of System Loads. AIEE Trans. 67 Part 2: 1474-1482. 1948.

4. Dahl, O. G. C. Electric Power Circuits. Vol. II. Power System Stability. N. Y., McGraw-Hill Book Co., Inc. 1938.

5. ________ and Fitzgerald, A. E. Equivalent Circuits in Stability Studies. AIEE Trans. 53: 1272-1282. 1934.

6. Falls, O. B. Jr. and Hobson, Charles. Equivalent Circuits for Representation of Large Metropolitan Area, Radial System, Loads and Generating Stations in Transient Stability Studies. Unpublished M. S. Thesis. Cambridge, Massachusetts, Massachusetts Institute of Technology Library. 1936.

7. Kimbark, E. W. Power System Stability. Vol. I. Elements of Stability Calculations. N. Y., John Wiley and Sons, Inc. 1948.

8. Maginnis, F. J. and Schultz, N. R. Transient Performance of Induction Motors. AIEE Trans. 63: 641-646. 1944.

9. Osterle, W. H. Reducing Load by Reducing Voltage. Electric Light and Power. 21: 56. 1943.

10. Park, R. H. and Bancker, E. H. System Stability as a Design Problem. AIEE Trans. 48: 170-193. 1929.

11. Puchstein, A. F., Lloyd, T. C. and Conrad, A. G. Alternating Current Machines. 3rd ed. N. Y., John Wiley and Sons, Inc. 1954.

12. Rüdenberg, Reinhold. Transient Performance of Electric Power Systems. N. Y., McGraw-Hill Book Co., Inc. 1950.

13. Shankle, D. F. and others. Transient Stability -- I. Synchronous and Induction Machines. AIEE Trans. 74 Part 3: 1563-1580. 1955.

14. Stanley, C. H. An Analysis of the Induction Motor. AIEE Trans. 57: 751-755. 1938.

15. Wagner, C. F. and Evans, R. D. Studies of Transmission Stability. AIEE Trans. 45: 51-94. 1926.

16. Westinghouse Electric and Manufacturing Co. Electric Transmission and Distribution Reference Book. 3rd ed. East Pittsburg, the Company. 1944.